SEM Power Estimates

Power : Probability of being able to reject H0 when H0 is false

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Approaches

- Rule of thumb - Subject : Variable
- Rule of thumb - Subject : Parameter
- Power estimates

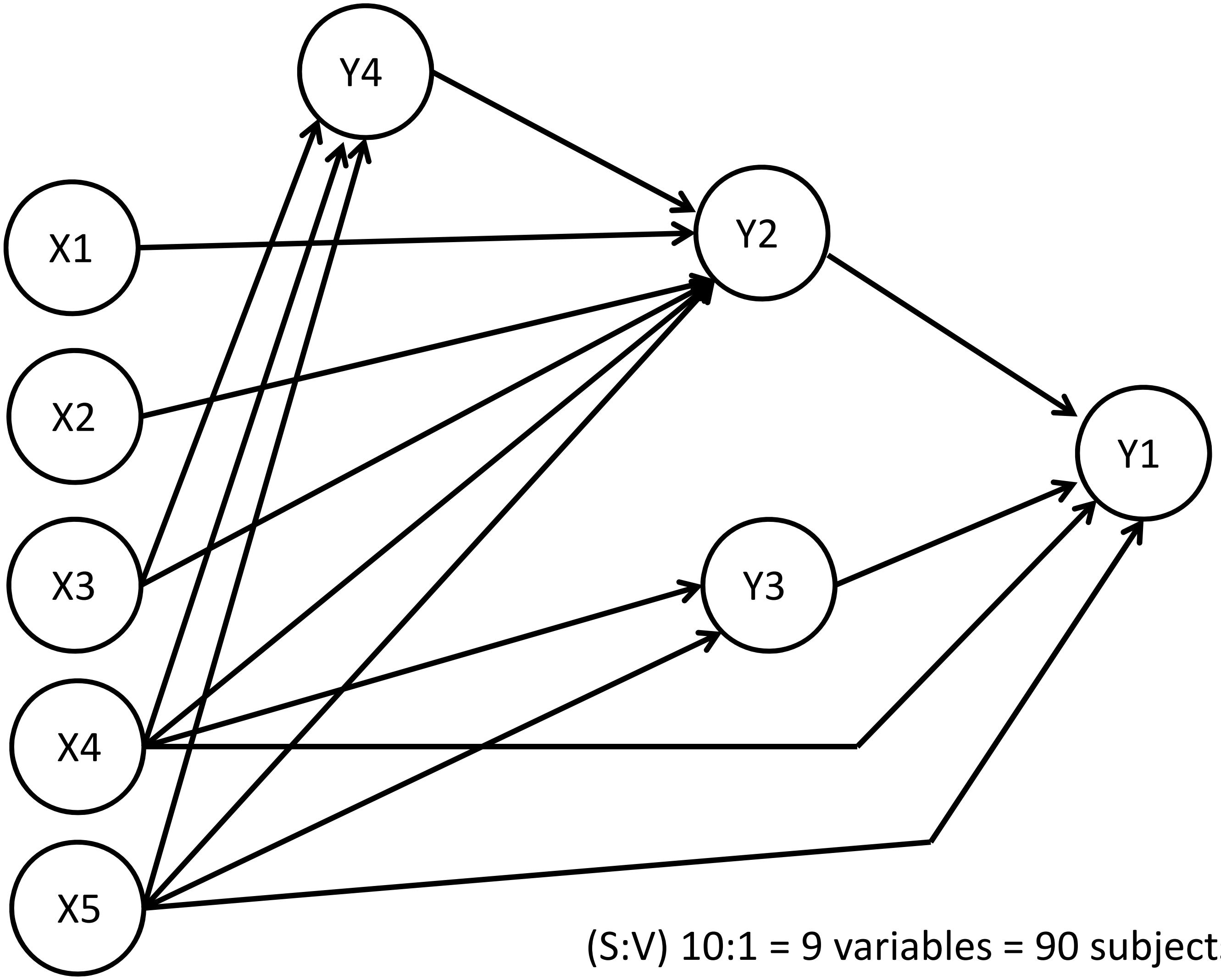
Power Estimate Approach

Satorra, A., and Saris, W. E. (1985). Power of the likelihood ratio test in covariance structure analysis. *Psychometrika*, 50, 83-90.

Power of the test (1 - b) for each path will be estimated using a procedure proposed by Satorra and Saris (1985) which approximates the noncentral chi-square distribution. Given this distribution it is possible to calculate the probability that the likelihood ratio statistic is larger than a certain value.

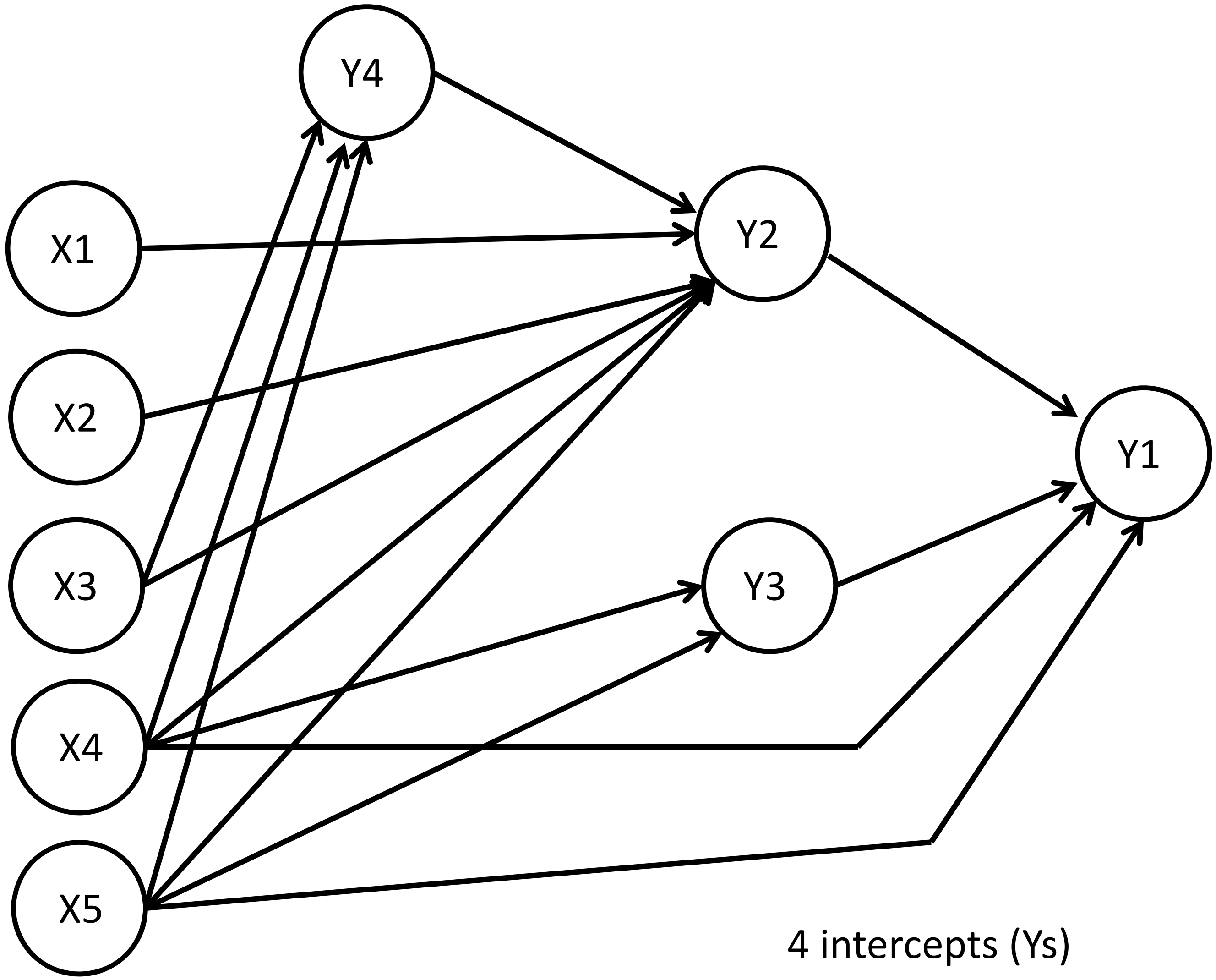
Satorra and Saris Approach

- H0: the model fits perfectly while H1: the model does not fit perfectly.
- Specify the reference model and determine the parameters of the model
- Obtain the expected covariance matrix ($\Sigma 0$)based on the reference model
- Fit the alternative model to the expected covariance matrix
- Obtain the fit function index and use it as non-centrality parameter
- Compute the probability of rejecting the null hypothesis: $F0=0$ (the alternative model fits perfectly)



(S:V) 10:1 = 9 variables = 90 subjects

(S:P) 10:1 = 23 free parameters = 230 subjects

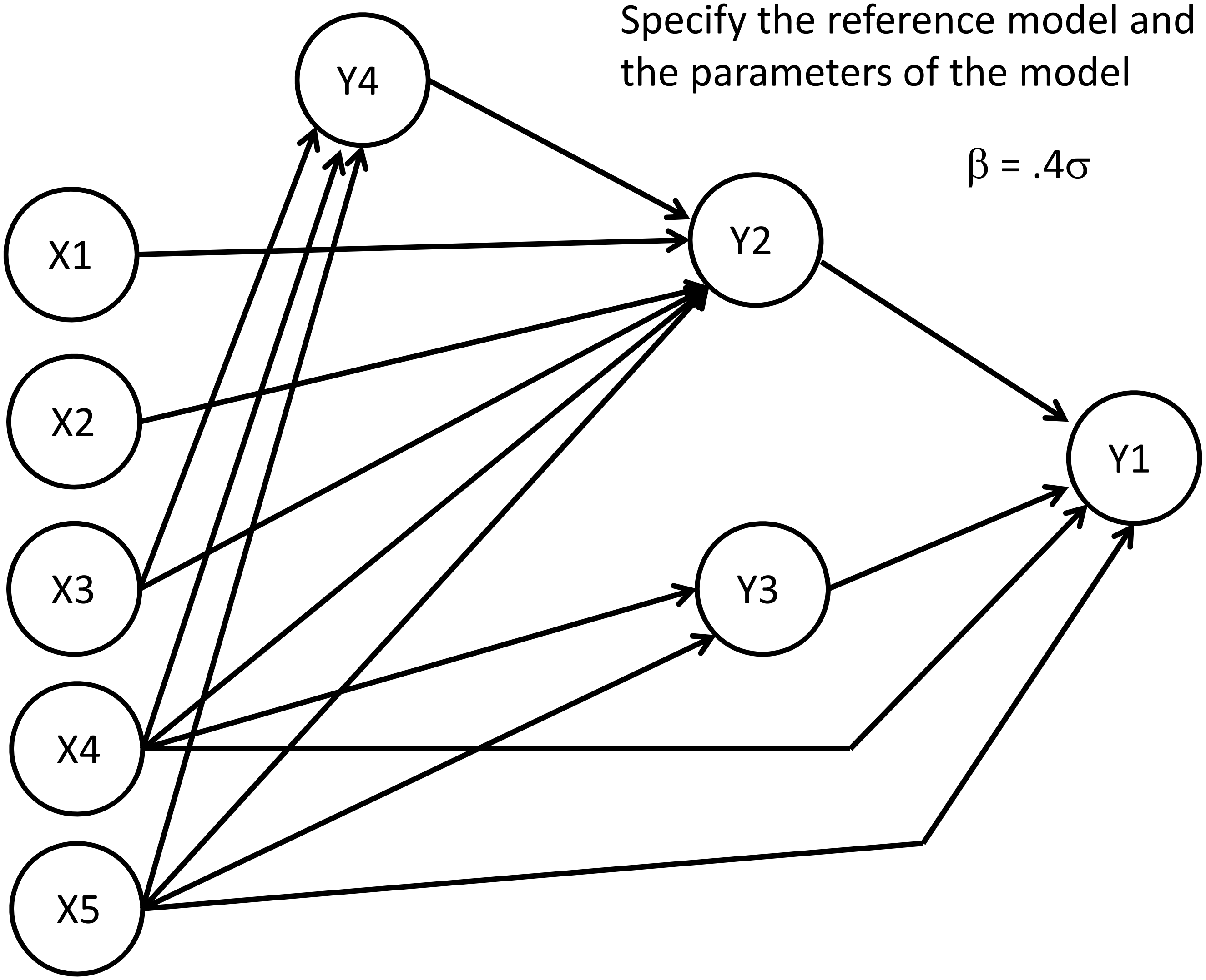


4 intercepts (Ys)
19 beta paths
4 residual variance estimates (Ys)
23 total parameters

STEP 1 Satorra and Saris Approach

Specify the reference model and determine the parameters of the model

$\beta = .4\sigma$



```
TITLE:    Power calculation for Intervention model
          Step 1: Computing the population means
          and covariance matrix

DATA:     FILE IS artific.dat;
          TYPE IS MEANS COVARIANCE;
          NOBSERVATIONS = 500;

VARIABLE: NAMES ARE y1-y4 x1-x5;

MODEL:
       y1 on y2@.4 y3@.4 x4@.4 x5@.4;
       y2 on x1@.4 x2@.4 y4@.4 x3@.4 x4@.4 x5@.4;
       y3 on x4@.4 x5@.4;
       y4 on x3@.4 x4@.4 x5@.4;

OUTPUT:  STANDARDIZED RESIDUAL tech4;
savedata:
     tech4 is pop4.dat;
```

Means →

```
0 0 0 0 0 0 0 0 0
1
0 1
0 0 1
0 0 0 1
0 0 0 0 1
0 0 0 0 0 1
0 0 0 0 0 0 1
0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 1
```

covariances →

```
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VARIABLE: NAMES ARE y1-y4 x1-x5;

MODEL:
       y1 on y2@.4 y3@.4 x4@.4 x5@.4;
       y2 on x1@.4 x2@.4 y4@.4 x3@.4 x4@.4 x5@.4;
       y3 on x4@.4 x5@.4;
       y4 on x3@.4 x4@.4 x5@.4;

OUTPUT:  STANDARDIZED RESIDUAL tech4;
savedata:
     tech4 is pop4.dat;
```

Obtain the expected covariance matrix (Σ0)based on the reference model

Population Matrix (Truth)

0.00000000E+00	0.00000000E+00	0.00000000E+00	0.00000000E+00	0.00000000E+00
0.00000000E+00	0.00000000E+00	0.00000000E+00	0.00000000E+00	
0.35263066E+01	0.20061849E+01	0.34505980E+01	0.11529175E+01	0.44710402E+00
0.16367897E+01	0.95169284E+00	0.12614720E+01	0.31936001E+00	0.19560800E+01
0.15968000E+00	0.39920001E+00	0.00000000E+00	0.00000000E+00	0.99800000E+00
0.15968000E+00	0.39920001E+00	0.00000000E+00	0.00000000E+00	0.00000000E+00
0.99800000E+00	0.22355201E+00	0.55888001E+00	0.00000000E+00	0.39920001E+00
0.00000000E+00	0.00000000E+00	0.99800000E+00	0.78243202E+00	0.55888001E+00
0.39920001E+00	0.39920001E+00	0.00000000E+00	0.00000000E+00	0.00000000E+00
0.99800000E+00	0.78243202E+00	0.55888001E+00	0.39920001E+00	0.39920001E+00
0.00000000E+00	0.00000000E+00	0.00000000E+00	0.00000000E+00	0.99800000E+00

Step 2
Checking (truth)

```
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          Step 1: Computing the population means
          and covariance matrix

DATA:     FILE IS artific.dat;
          TYPE IS MEANS COVARIANCE;
          NOBSERVATIONS = 500;

VARIABLE: NAMES ARE y1-y4 x1-x5;

MODEL:
       y1 on y2@.4 y3@.4 x4@.4 x5@.4;
       y2 on x1@.4 x2@.4 y4@.4 x3@.4 x4@.4 x5@.4;
       y3 on x4@.4 x5@.4;
       y4 on x3@.4 x4@.4 x5@.4;

OUTPUT:  STANDARDIZED RESIDUAL tech4;
savedata:
     tech4 is pop4.dat;
```

```
TITLE:    Power calculation for Intervention model
          Step 2: Check population means
          and covariance matrix

DATA:     FILE IS pop4.dat;
          TYPE IS MEANS COVARIANCE;
          NOBSERVATIONS = 500;

VARIABLE: NAMES ARE y1-y4 x1-x5;


MODEL:
       y1 on y2 y3 x4 x5;
       y2 on x1 x2 y4 x3 x4 x5;
       y3 on x4 x5;
       y4 on x3 x4 x5;

OUTPUT:  tech1;
```

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Y1 ON				
Y2	0.400	0.034	11.750	0.000
Y3	0.400	0.050	8.024	0.000
X4	0.400	0.064	6.292	0.000
X5	0.400	0.064	6.292	0.000
Y2 ON				
X1	0.400	0.063	6.389	0.000
X2	0.400	0.063	6.389	0.000
Y4	0.400	0.051	7.772	0.000
X3	0.400	0.066	6.069	0.000
X4	0.400	0.066	6.069	0.000
X5	0.400	0.066	6.069	0.000
Y3 ON				
X4	0.400	0.051	7.785	0.000
X5	0.400	0.051	7.785	0.000
Y4 ON				
X3	0.400	0.054	7.352	0.000
X4	0.400	0.054	7.352	0.000
X5	0.400	0.054	7.352	0.000
Intercepts				
Y1	0.000	0.057	0.000	1.000
Y2	0.000	0.062	0.000	1.000
Y3	0.000	0.051	0.000	1.000
Y4	0.000	0.054	0.000	1.000
Residual Variances				
Y1	1.634	0.103	15.811	0.000
Y2	1.952	0.123	15.811	0.000
Y3	1.315	0.083	15.811	0.000
Y4	1.474	0.093	15.811	0.000

Step 3

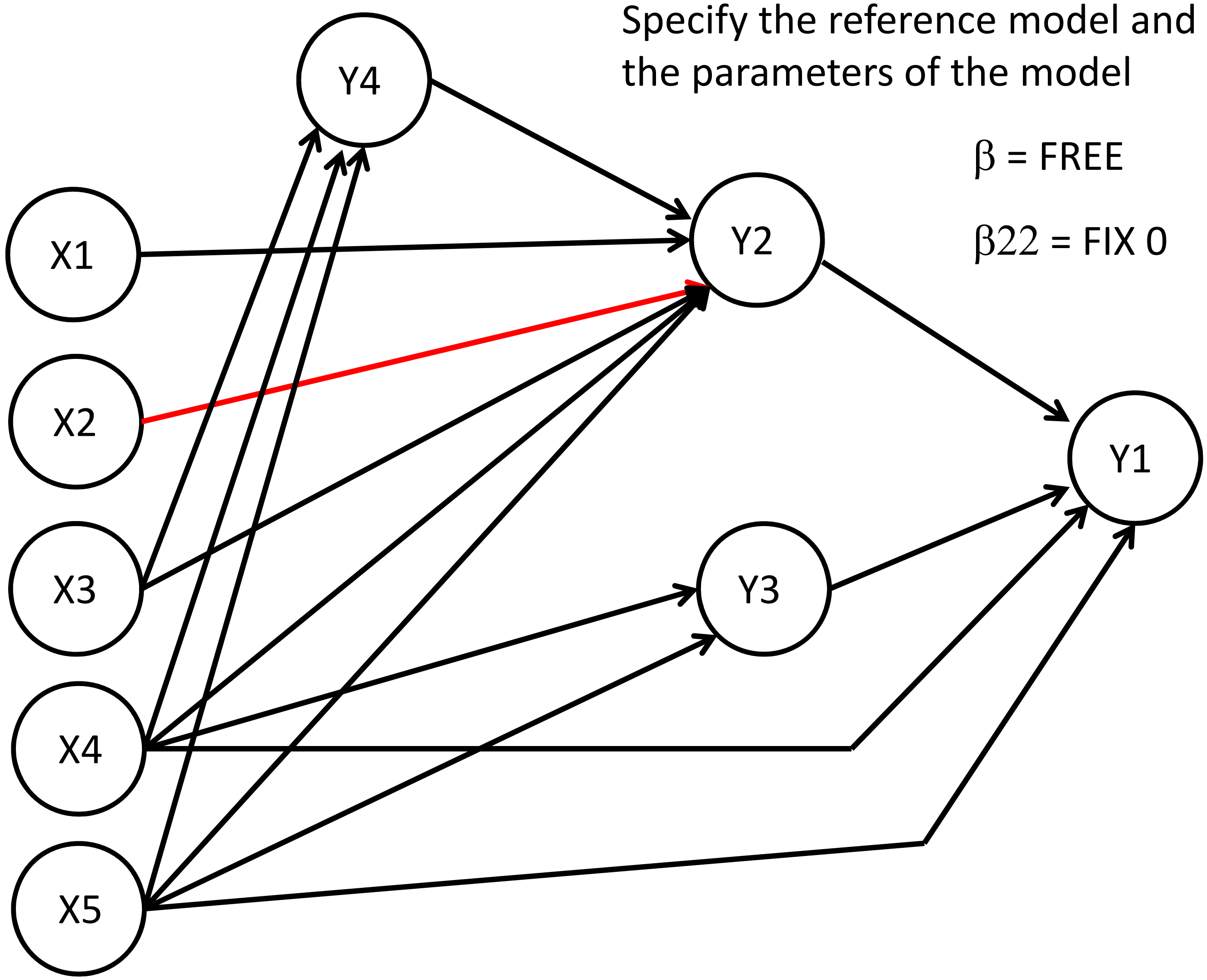
- Fit the alternative model to the expected covariance matrix
- Obtain the fit function index and use it as non-centrality parameter to compute the probability of rejecting the null hypothesis

STEP 1 Satorra and Saris Approach

Specify the reference model and determine the parameters of the model

β = FREE

$\beta 22$ = FIX 0



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TITLE:    Power calculation for Intervention model
          Step 3: Estimate power for path

DATA:     FILE IS pop4.dat;
          TYPE IS MEANS COVARIANCE;
          NOBSERVATIONS = 100;

VARIABLE: NAMES ARE y1-y4 x1-x5;


MODEL:
       y1 on y2 y3 x4 x5;
       y2 on x1 x2@0 y4 x3 x4 x5;
       y3 on x4 x5;
       y4 on x3 x4 x5;

OUTPUT:  STANDARDIZED RESIDUAL;
```

We can manipulate the sample size

Now number of free parameters = 22 not 23

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Y1 ON				
Y2	0.400	0.076	5.255	0.000
Y3	0.400	0.111	3.588	0.000
X4	0.400	0.142	2.814	0.005
X5	0.400	0.142	2.814	0.005
Y2 ON				
X1	0.400	0.146	2.747	0.006
X2	0.000	0.000	999.000	999.000
Y4	0.400	0.120	3.342	0.001
X3	0.400	0.153	2.610	0.009
X4	0.400	0.153	2.610	0.009
X5	0.400	0.153	2.610	0.009
Y3 ON				
X4	0.400	0.115	3.481	0.000
X5	0.400	0.115	3.481	0.000
Y4 ON				
X3	0.400	0.122	3.288	0.001
X4	0.400	0.122	3.288	0.001
X5	0.400	0.122	3.288	0.001
Intercepts				
Y1	0.000	0.127	0.000	1.000
Y2	0.000	0.145	0.000	1.000
Y3	0.000	0.114	0.000	1.000
Y4	0.000	0.121	0.000	1.000
Residual Variances				
Y1	1.620	0.229	7.071	0.000
Y2	2.095	0.296	7.071	0.000
Y3	1.304	0.184	7.071	0.000
Y4	1.462	0.207	7.071	0.000

Step 2

```
Chi-Square Test of Model Fit

          Value                              0.000
          Degrees of Freedom                    11
          P-Value                           1.0000
```

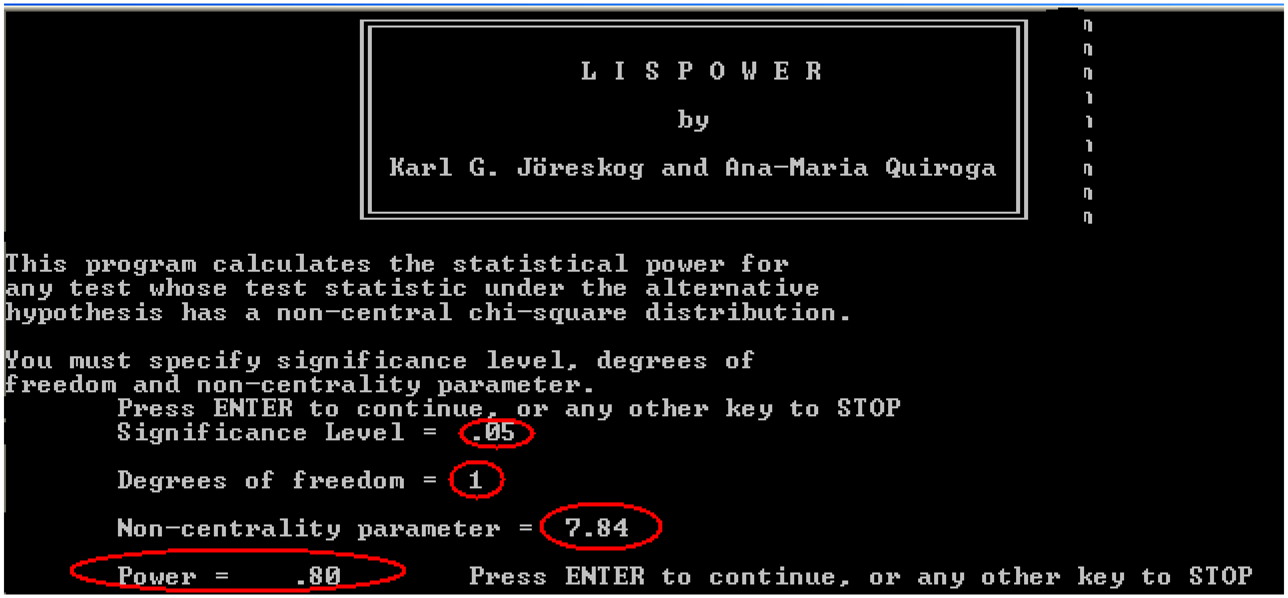
Step 3

```
TESTS OF MODEL FIT

Chi-Square Test of Model Fit

          Value                              7.847
          Degrees of Freedom                    12
          P-Value                           0.7969
```

Non-centrality parameter = $\chi 2$ with 1 df



For a sampling of 100 in this structure, the power to detect a .40σ for β22 is 0.80.