Structural Equation Modeling Workshop

PIRE

August 6-7, 2007

Section 1

Introduction to SEM

Definitions of Structural Equation Models/Modeling

- "Structural equation modeling (SEM) does not designate a single statistical technique but instead refers to a family of related procedures. Other terms such as covariance structure analysis, covariance structural modeling, or analysis of covariance structures are essentially interchangeable. Another term…is causal modeling, which is used mainly in association with the technique of path analysis. This expression may be somewhat dated, however, as it seems to appear less often in the literature nowadays." (Kline, 2005)

History of SEM

- Sewall Wright and Path Analysis
- Duncan and Path Analysis
- Econometrics
- Joreskog and LISREL
- Bentler and EQS
- Muthen and Mplus

Sewall Wright

- Geneticist
- Principle of Path Analysis provides algorithm for decomposing correlations of 2 variables into structural relations among a set of variables
- Created the path diagram
- Applied path analysis to genetics, psychology, and economics

Duncan

- Applied path analysis methods to the area of social stratification (occupational attainment)
- Key papers by Duncan & Hodge (1964) and Blau & Duncan (1967)
- Developed one of the first texts on path analysis

Econometrics

- Goldberger added the importance of standard errors and links to statistical inference
- Showed how ordinary least squares estimates of parameters in overidentified systems of equations were more efficient than averages of multiple estimates of parameters
- Combined psychometric and econometric components

Indirect Effects

- Duncan (1966, 1975)—applying tracing rules
- Reduced-form equations (Alwin & Hauser, 1975)
- Asymptotic distribution of indirect effects (Sobel, 1982)

Joreskög

- Maximum Likelihood estimator was an improvement over 2 and 3 stage least squares methods
- Joreskög made structural equation modeling more accessible (if only slightly!) with the introduction of LISREL, a computer program
- Added model fit indices
- Added multiple-group models

Bentler

- Refined fit indices
- Added specific effects and brought SEM into the field of psychology, which otherwise was later than economics and sociology in its introduction to SEM

Muthén

- Added latent growth curve analysis
- Added hierarchical (multi-level) modeling

Other Developments

- Models for dichotomous and ordinal variables
- Various combinations of hierarchical (multi-level) modeling, latent growth curve analysis, multiple-group analyses
- Use of interaction terms

Quips and Quotes (Wolfle, 2003)

- "Here I was doing elaborate, cross-lagged, multiple-partial canonical correlations involving dozens of variables, and that eminent sociologist [Paul Lazarsfeld] was still messing around with chi square tables! What I did not appreciate was that his little analyses were generally more informative than my elaborate ones, because he had the 'right' variables. He knew his subject matter. He was aware of the major alternative explanations that had to be guarded against and took that into account when he decided upon the four or five variables that were crucial to include. His work represented the state of the art in model building, while my work represented the state of the art in number crunching." (Cooley, 1978)

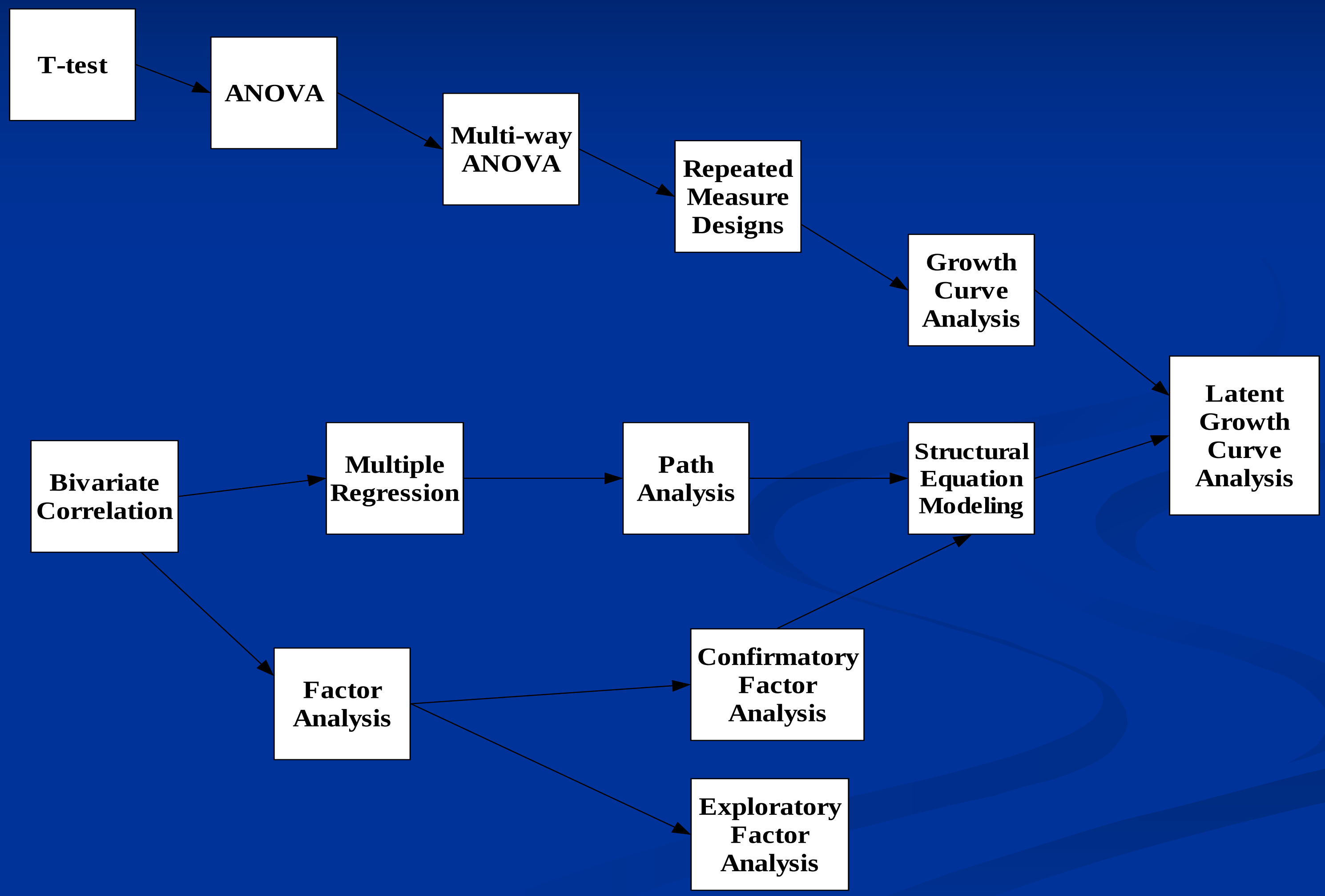
Quips and Quotes (cont.)

- "All models are wrong, but some are useful." (Box, 1979)
- "Analysis of covariance structures…is explicitly aimed at complex testing of theory, and superbly combines methods hitherto considered and used separately. It also makes possible the rigorous testing of theories that have until now been very difficult to test adequately." (Kerlinger, 1977)

Quips and Quotes (cont.)

- "The government are very keen on amassing statistics. They collect them, add them, raise them to the nth power, take the cube root and prepare wonderful diagrams. But you must never forget that every one of these figures come in the first instance from the village watchman, who just puts down what he damn pleases." (Sir J. Stamp, 1929)

Family Tree of SEM



Defining SEM

- "a melding of factor analysis and path analysis into one comprehensive statistical methodology" (Kaplan, 2000)
- Simultaneous equation modeling
- Does implied covariance matrix match up with observed covariance matrix
- Degree to which they match represents goodness of fit

Types of SEM Models

- Path Analysis Models
- Confirmatory factory analysis models
- Structural regression models
- Latent change models

How SEM and traditional approaches are different

- Multiple equations can be estimated simultaneously
- Non-recursive models are possible
- Correlations among disturbances are possible
- Formal specification of a model is required
- Measurement and structural relations are separated, with relations among latent variables rather than measured variables
- Assessing of model fit is not as straightforward

Why Use SEM?

- Test full theoretical model
  - ELM as argued by Stiff & Mongeau (1993)
- Simultaneous (full information) estimation
  - consistent with SEM statistical theory
  - Analyze systems of equations
  - Assumptions about data distribution
  - But…error spread throughout model
- Latent Variables
  - Divorce measurement error
  - True systematic relationship between variables

Ways to Increase Confidence in Causal Explanations

- Conduct experiment if possible
- If not:
  - Control for additional potential confounding (independent or mediating) variables
  - Control for measurement error (as in SEM)
  - Make sure statistical power is adequate to detect effects or test model
  - Use theory, carefully conceptualize variables, and carefully select variables for inclusion
  - Compare models rather than merely assessing one model
  - Collect data longitudinally if possible

Section 2:
Review of Correlation and Regression

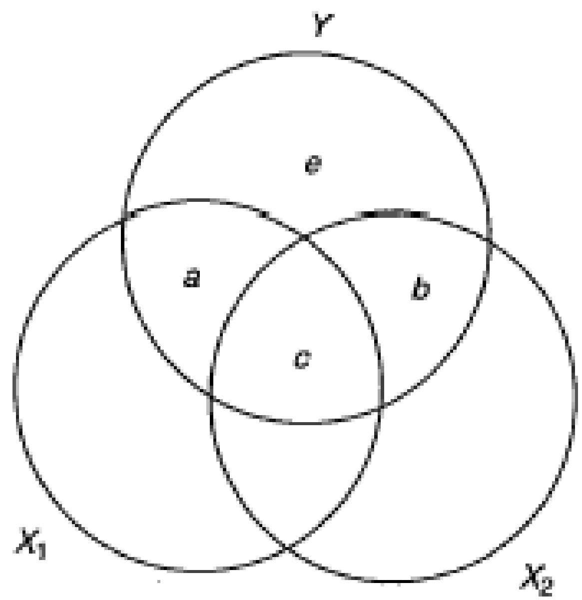
Factors Affecting the size of r

- Arithmetic operations: generally no effect
- Distributions of X and Y
- Reliability of variables
- Restriction of range

Definitions of semi-partial and partial correlation coefficients

- Correlation between Y and X_1 where effects of X_2 have been removed from X_1 but not from Y is semi-partial correlation (a or b in the Venn Diagram)
- Squared partial correlation answers the question, How much of Y that is not estimated by the other IVs is estimated by this variable? a/(a+e) or b/(b+e)

Components of Explained Variance in 2-independent variable Case



$$r^2_{Y1} = a + c$$

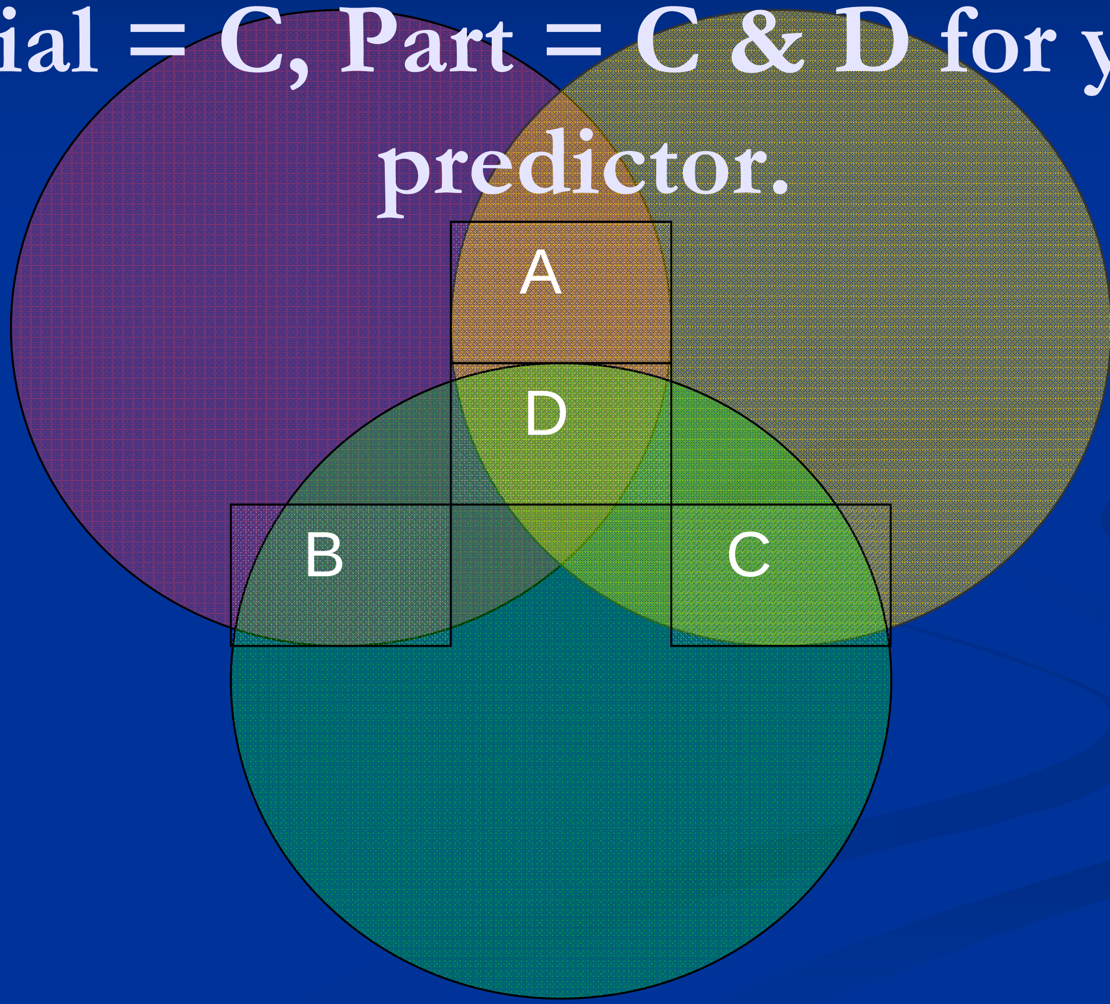
$$r^2_{Y2} = b + c$$

$$R^2_{Y.12} = a + b + c$$

FIGURE 3.3.1 The ballantine for X_1 and X_2 with Y.

Partial = B, Part = B & D for purple predictor.

Partial = C, Part = C & D for yellow predictor.



Interpretation of Part Correlations

1. Part correlation (semi partial) squared is the unique amount of total variance explained.
2. Sum of part correlations squared does NOT equal R^2 because of overlapping variance.
3. The part correlation^2 does tell you how much R^2 would decrease if that predictor was eliminated.

Ways to account for shared variance

- A Partial regression coefficient is the correlation between a specific predictor and the criterion when statistical control has occurred for all other variables in the analysis, meaning all the variance for the other predictors is completely removed.
- A Part (semi partial) regression coefficient is the correlation between a specific predictor and the criterion when all other predictors have been partialed out of that predictor, but not out of the criterion.

Possible Relationships among Variables

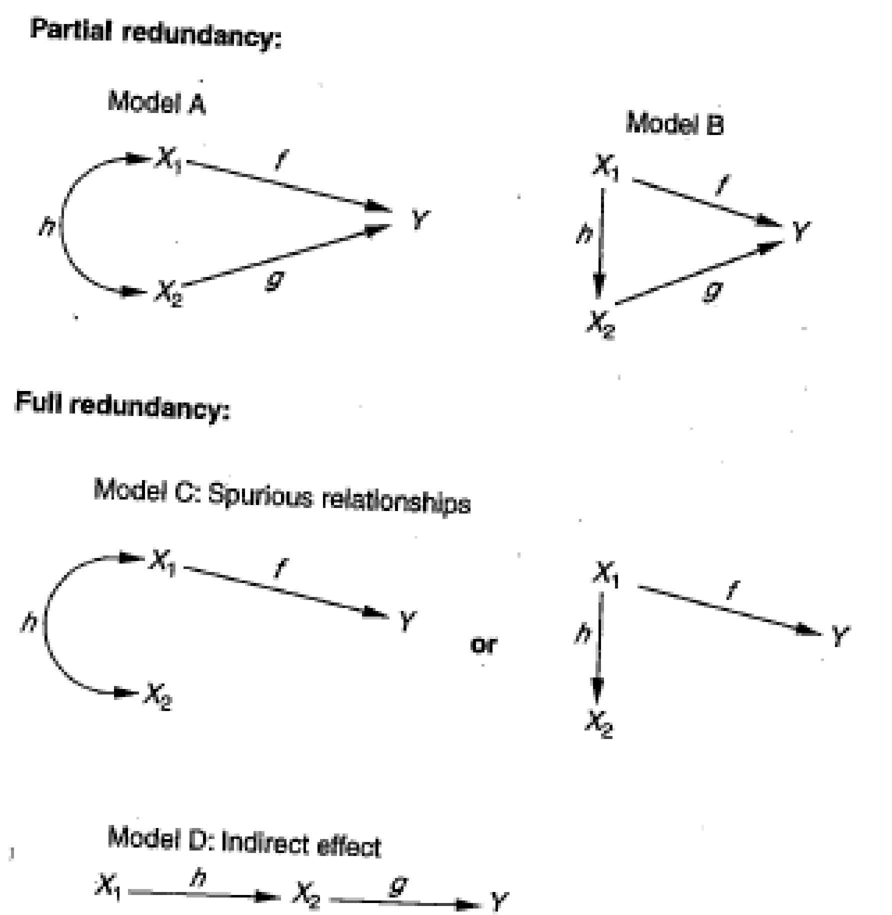


FIGURE 3.4.1 Representation of relationships between Y and two IVs.

Suppression

- The relationship between the independent or causal variables is hiding or suppressing their real relationships with Y, which would be larger or possibly of opposite sign were they not correlated.
- The inclusion of the suppressor in the regression equation removes the unwanted variance in X_1 in effect enhanced the relationship between X_1 and Y.

Effects of Specification Error

- Specification Error when variables are omitted from the regression equation
- Effects can be inflated or diminished regression coefficients of the variables in the model, and a reduced R^2

Multicollinearity

- Existence of substantial correlation among a set of independent variables.
- Problems of interpretation and unstable partial regression coefficients

Section 3

Data Screening: Fixing Distributional Problems, Missing Data, Measurement

Multicollinearity

- Existence of substantial correlation among a set of independent variables.
- Problems of interpretation and unstable partial regression coefficients
- Tolerance = 1 – R^2 of X with all other X
- VIF = 1/Tolerance
- VIF < 8.0 not a bad indicator
- How to fix:
  - Delete one or more variables
  - Combine several variables

Standardized vs. Unstandardized Regression Coefficients

- Standardized coefficients can be compared across variables within a model
- Standardized coefficients reflect not only the strength of the relationship but also variances and covariances of variables included in the model as well of variance of variables not included in the model and subsumed under the error term
- As a result, standardized coefficients are sample-specific and cannot be used to generalize across settings and populations

Standardized vs. Unstandardized Regression Coefficients (cont.)

- Unstandardized coefficients, however, remain fairly stable despite differences in variances and covariances of variables in different settings or populations
- A recommendation: Use std. coeff. to compare effects within a given population, but unstd. coeff. to compare effects of given variables across populations.
- In practice, when units are not meaningful, behavioral scientists outside of sociology and economics use standardized coefficients in both cases.

Fixing Distributional Problems

- Analyses assume normality of individual variables and multivariate normality, linearity, and homoscedasticity of relationships
- Normality: similar to normal distribution
- Multivariate normality: residuals of prediction are normally and independently distributed
- Homoscedasticity: Variances of residuals do not vary across values of X

Transformations: Ladder of Re-Expressions

- Power
- Inverses (roots)
- Logarithms
- Reciprocals

Suggested Transformations

Distributional Problem	Transformation
Mod. Pos. Skew	Square root
Substantial pos. skew	Log (x+c)*
Severe pos. skew, L-shaped	1/(x+c)*
Mod. Negative skew	Square root (k-x)
Substantial neg. skew	Log (k-x)
Severe. Neg. skew, J shaped	1/(k-x)

Dealing with Outliers

- Reasons for univariate outliers:
  - Data entry errors--correct
  - Failure to specify missing values correctly--correct
  - Outlier is not a member of the intended population--delete
  - Case is from the intended population but distribution has more extreme values than a normal distribution—modify value
  - 3.29 or more SD above or below the mean a reasonable dividing line, but with large sample sizes may need to be less inclusive

Multivariate outliers

- Cases with unusual patterns of scores
- Discrepant or mismatched cases
- Mahalanobis distance: distance in SD units between set of scores for individual case and sample means for all variables

Linearity and Homoscedasticity

- Either transforming variable(s) or including polynomial function of variables in regression may correct linearity problems
- Correcting for normality of one or more variables, or transforming one or more variables, or collapsing among categories may correct heteroscedasticity. "Not fatal," but weakens results.

Missing Data

- How much is too much?
  - Depends on sample size
  - 20%?
- Why a problem?
  - Reduce power
  - May introduce bias in sample and results

Types of Missing Data Patterns

- Missing at random (MAR)—missing observations on some variable X differ from observed scores on that variable only by chance. Probabilities of missingness may depend on observed data but not missing data.
- Missing completely at random (MCAR)—in addition to MAR, presence vs. absence of data on X is unrelated to other variables. Probabilities of missingness also not dependent on observed ata.
- Missing not at random (MNAR)

Methods of Reducing Missing Data

- Case Deletion
- Substituting Means on Valid Cases
- Substituting estimates based on regression
- Multiple Imputation
  - Each missing value is replaced by list of simlulated values. Each of m datasets is analyzed by a complete-data method. Results combined by averaging results with overall estimates and standard errors.
- Maximum Likelihood (EM) method:
  - Fill in the missing data with a best guess under current estimate of unknown parameters, then reestimate from observed and filled-in data

Checklist for Screening Data

- Inspect univariate descriptive statistics
- Evaluate amount/distribution of missing data
- Check pairwise plots for nonlinearity and heteroscedasticity
- Identify and deal with nonnormal variables
- Identify and deal with multivariate outliers
- Evaluate variables for multicollinearity
- Assess reliability and validity of measures

Section 4

Overview of SEM concepts, path diagrams, programs

Definitions

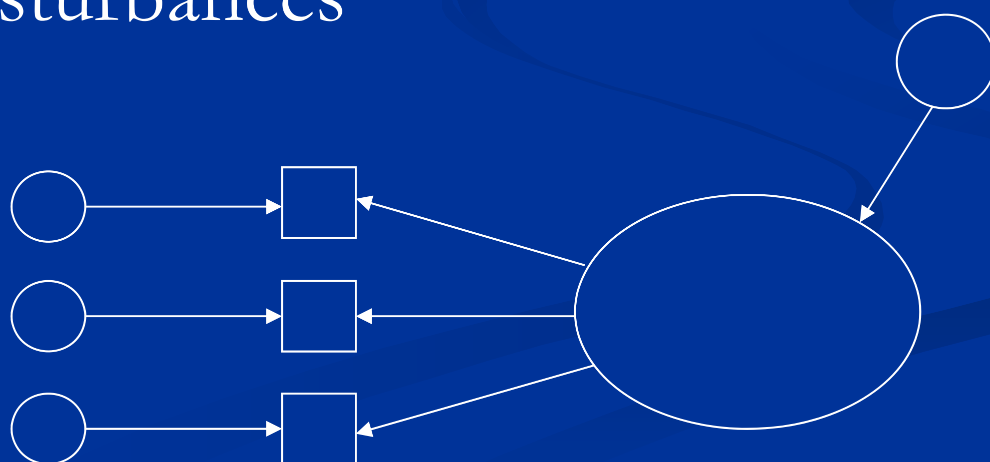
- Exogenous variable—Independent variables not presumed to be caused by variables in the model
- Endogenous variables— variables presumed to be caused by other variables in the model
- Latent variable: unobserved variable implied by the covariances among two or more indicators, free of random error (due to measurement) and uniqueness associated with indicators, measure of theoretical construct
- Measurement model prescribes components of latent variables
- Structural model prescribes relations among latent variables and/or observed variables not linked to latent variables
- Recursive models assume that all causal effects are represented as unidirectional and no disturbance correlations among endogenous variables with direct effects between them
- Non-recursive models are those with feedback loops

Definitions (cont.)

- Model Specification—Formally stating a model via statements about a set of parameters
- Model Identification—Can a single unique value for each and every free parameter be obtained from the observed data: just identified, over-identified, under-identified
- Evaluation of Fit—Assessment of the extent to which the overall model fits or provides a reasonable estimate of the observed data
- Fixed (not estimated, typically set = 0), Free (estimated from the data), and Constrained Parameters (typically set of parameters set to be equal)
- Model Modification—adjusting a specified and estimated model by freeing or fixing new parameters
- Direct (presumed causal relationship between 2 variables), indirect (presumed causal relationship via other intervening or mediating variables), and total effects (sum of direct and indirect effects)

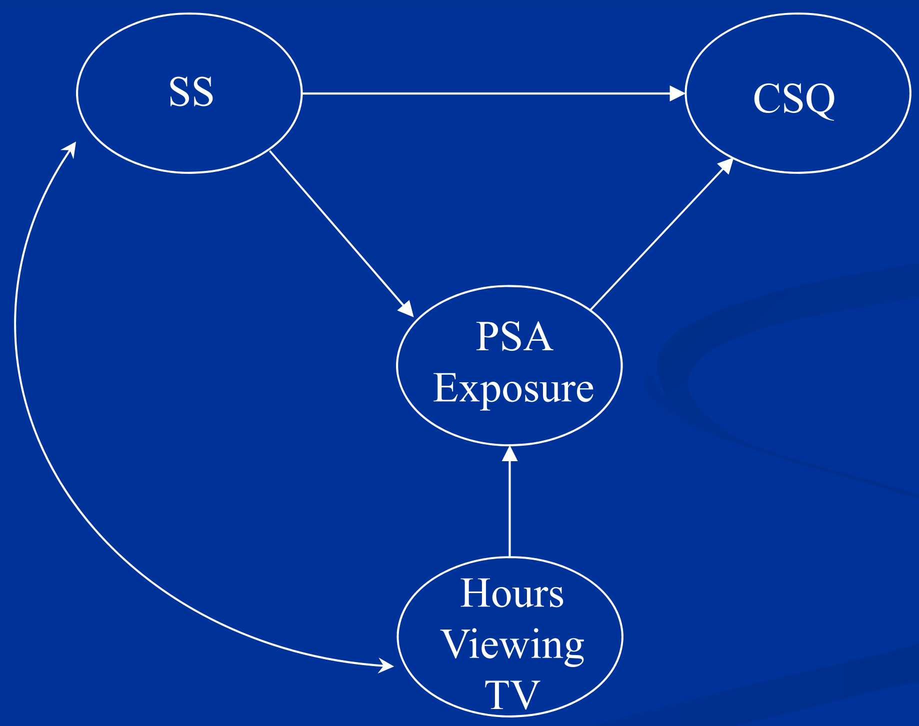
Path Diagrams

- Ovals for latent variables
- Rectangles for observed variables
- Arrows point toward observed variables to indicate measurement error
- Arrows point toward latent variables to indicate residuals or disturbances



Path Diagrams

- Straight lines for putative causal relations
- Curved lines to indicate correlations



Confirmatory Factor Analysis

- The concept and practice of what most of us know as factor analysis is now considered *exploratory factor analysis*, that is, with no or few preconceived notions about what the factor pattern will look like. There are typically no tests of significance for EFA
- *Confirmatory factory analysis*, on the other hand, is where we have a theoretically or empirically based conception of the structure of measured variables and factors and enables us to test the adequacy of a particular "measurement model" to the data

Structural Regression Models

- Inclusion of measured and latent variables
- Assessment both of relationship between measured and latent variables (measurement model) and putative causal relationships among latent variables (structural model)
- Controls for measurement error, correlations due to methods, correlations among residuals and separates these from structural coefficients

Path Diagrams

- Ovals for latent variables
- Rectangles for observed variables
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- Arrows pointing toward observed variables to indicate measurement error
- Arrows pointing toward latent variables to indicate residuals or disturbances

Steps in SEM

- Specify the model
- Determine identification of the model
- Select measures and collect, prepare and screen the data
- Use a computer program to estimate the model
- Re-specify the model if necessary
- Describe the analysis accurately and completely
- Replicate the results*
- Apply the results*

Programs

- AMOS—assess impact of one parameter on model; editing/debugging functions; bootstrapped estimates; MAR estimates
- EQS—data editor; wizard to write syntax; various estimates for nonnormal data; model-based bootstrapping and handling randomly missing data
- LISREL—data entry to analysis. PRELIS screens data files; wizard to write syntax; can easily analyze categorical/ordinal variables; hierarchical data can also be used
- MPLUS—latent growth models; wizard for batch analysis; no model diagram input/output; MAR data; complex sampling designs; hierarchical and multi-level models

Section 5

Equations for path analysis, decomposing correlations, mediation

Path Equations

- Components of Path Model:
  - Exogenous Variables
  - Correlations among exogenous variables
  - Structural paths
  - Disturbances/residuals/error

Relationship between regression coefficients and path coefficients

- When residuals are uncorrelated with variables in the equation in which it appears, nor with any of the variables preceding it in the model, the solution for the path coefficients takes the form of OLS solutions for the standardized regression coefficients.

The Tracing Rule

- If one causes the other, then always start with the one that is the effect. If they are not directly causally related, then the starting point is arbitrary. But once a start variable is selected, always start there.
- Start against an arrow (go from effect to cause). Remember, the goal at this point is to go from the start variable to the other variable.
- Each particular tracing of paths between the two variables can go through only one noncausal (curved, double-headed) path (relevant only when there are three or more exogenous variables and two or more curved, double-headed arrows).

The Tracing Rule (cont.)

- For each particular tracing of paths, any intermediate variable can be included only once.
- The tracing can go back against paths (from effect to cause) for as far as possible, but, regardless of how far back, once the tracing goes forward causally (i.e., with an arrow from cause to effect), it cannot turn back against an arrow.

Mediation vs. Moderation

- Mediation: Intervening variables
- Moderation: Interaction among independent or interventing/mediating variables

How to Test for Mediation

- X → Y
- X → M
- M → Y
- When M is added to X as predictor of Y, X is no longer significantly predictive of Y (Baron & Kenny)
- Assess effect ratio: a X b / c [indirect effect divided by direct effect]

Direct, Indirect, and Total Effects

- Total Effect = Direct + Indirect Effects
- Total Effect = Direct Effects + Indirect Effects + Spurious Causes + Unanalyzed due to correlated causes

Identification

- A model is identified if:
  - It is theoretically possible to derive a unique estimate of each parameter
  - The number of equations is equal to the number of parameters to be estimated
  - It is fully recursive

Overidentification

- A model is overidentified if:
  - A model has fewer parameters than observations
  - There are more equations than are necessary for the purpose of estimating parameters

Underidentification

- A model is underidentified or not identified if:
  - It is not theoretically possible to derive a unique estimate of each parameter
  - There is insufficient information for the purpose of obtaining a determinate solution of parameters.
  - There are an infinite number of solutions may be obtained

Necessary but not Sufficient Conditions for Identification: Counting Rule

- Counting rule: Number of estimated parameters cannot be greater than the number of sample variances and covariances. Where the number of observed variables = p, this is given by

  $[p \times (p+1)] / 2$

Necessary but not Sufficient Conditions for Identification: Order Condition

- If m = # of endogenous variables in the model and k = # of exogenous variables in the model, and k_e = # exogenous variables in the model excluded from the structural equation model being tested and m_i = number of endogenous variables in the model included in the equation being tested (including the one being explained on the left-hand side), the following requirement must be satisfied: $k_e \geq m_i - 1$

Necessary but not Sufficient Conditions for Identification: Rank Condition

- For nonrecursive models, each variable in a feedback loop must have a unique pattern of direct effects on it from variables outside the loop.
- For recursive models, an analogous condition must apply which requires a very complex algorithm or matrix algebra.

Guiding Principles for Identification

- A fully recursive model (one in which all the variables are interconnected) is just identified.
- A model must have some scale for unmeasured variables

Where are Identification Problems More Likely?

- Models with large numbers of coefficients relative to the number of input covariances
- Reciprocal effects and causal loops
- When variance of conceptual level variable and all factor loadings linking that concept to indicators are free
- Models containing many similar concepts or many error covariances

How to Avoid Underidentification

- Use only recursive models
- Add extra constraints by adding indicators
- Fixed whatever structural coefficients are expected to be 0, based on theory, especially reciprocal effects, where possible
- Fix measurement error variances based on known data collection procedures
- Given a clear time order, reciprocal effects shouldn't be estimated
- If the literature suggests the size of certain effects, one can fix the coefficient of that effect to that constant

How to Test for Underidentification

- If ML solution repeatedly converges to same set of final estimates given different start values, suggests identification
- If concerned about the identification of a particular equation/coefficient, run the model once with the coefficient free, once at a value thought to be "minimally yet substantially different" than the estimated value. If the fit of the model is worse, it suggests identification.

What to do if a Model is Underidentified

- Simplify the model
- Add indicators
- Eliminate reciprocal effects
- Eliminate correlations among residuals

Introduction to AMOS,

Part 1

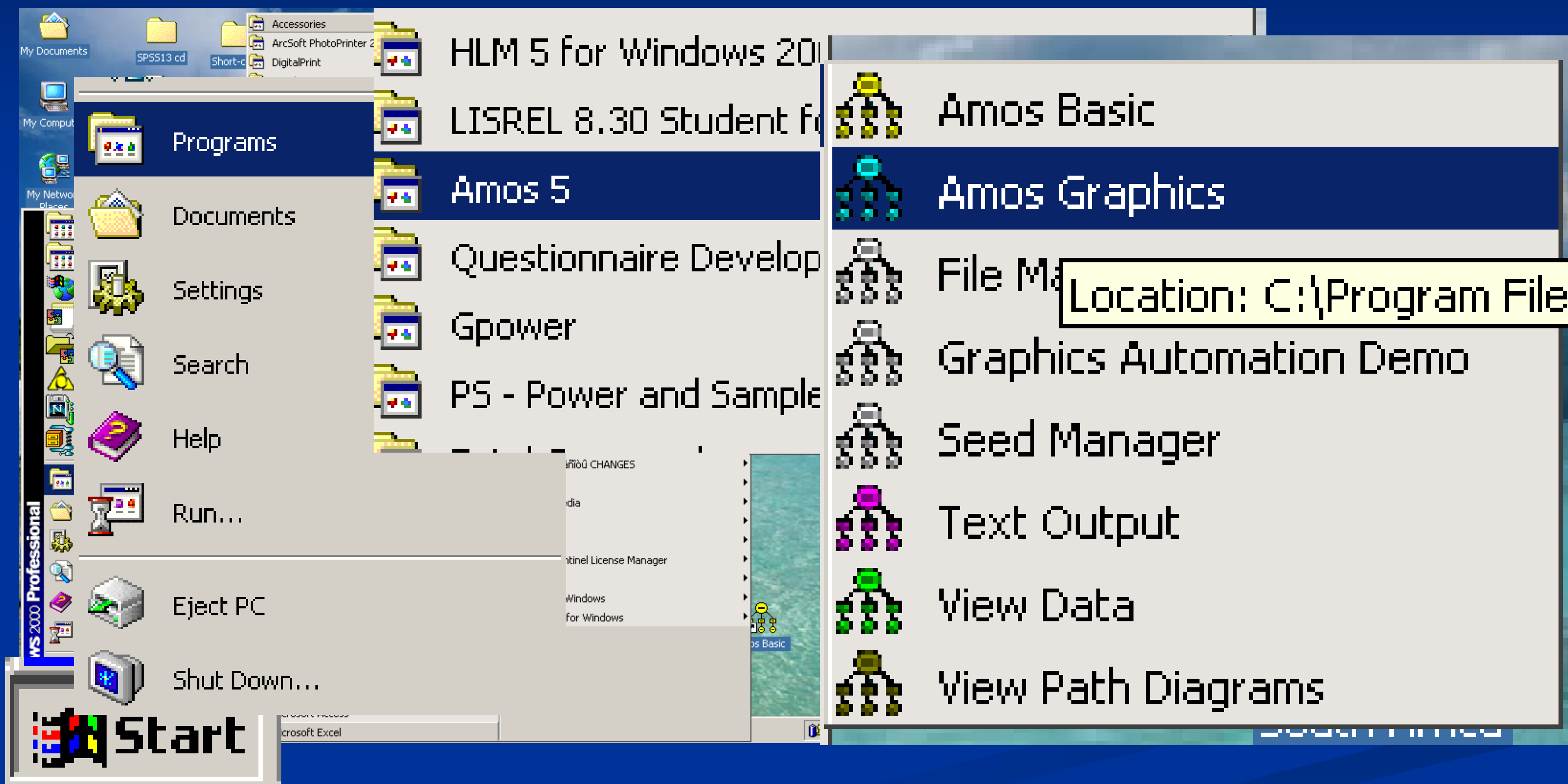
AMOS Advantages

- Easy to use for visual SEM (Structural Equation Modeling).
- Easy to modify, view the model
- Publication –quality graphics

AMOS Components

- AMOS Graphics
  - draw SEM graphs
  - runs SEM models using graphs
- AMOS Basic
  - runs SEM models using syntax

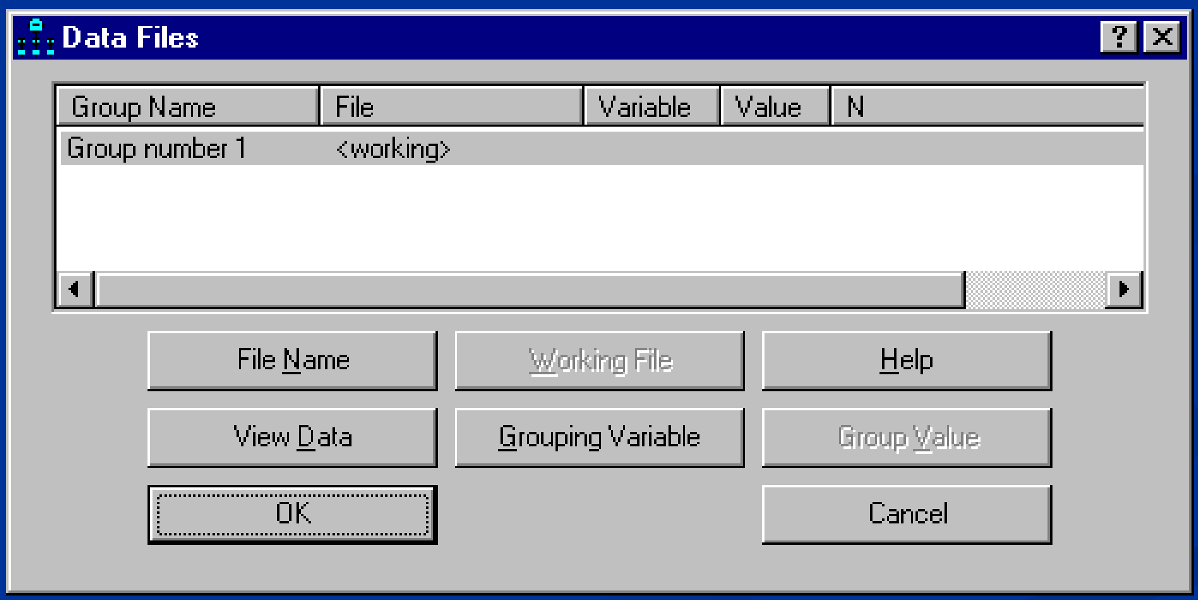
Starting AMOS Graphics



Start → Programs → Amos 5 → Amos Graphics

Reading Data into AMOS

- File → Data Files
- The following dialog appears:



Reading Data into AMOS

- Click on File Name to specify the name of the data file

◆ Currently AMOS reads the following data file formats:

- Access
- dBase 3 – 5
- Microsft Excel 3, 4, 5, and 97
- FoxPro 2.0, 2.5 and 2.6
- Lotus wk1, wk3, and wk4
- **SPSS *.sav files**, versions 7.0.2 through 13.0 (both raw data and matrix formats)

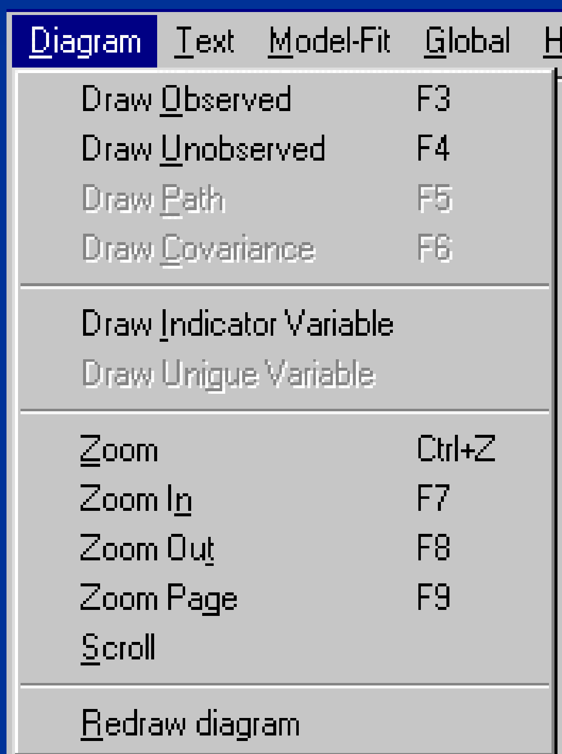
Reading Data into AMOS

- Example USED for this workshop:
  - Condom use and what predictors affect it

- DATASET: **AMOS_data_valid_condom.sav**

Drawing in AMOS

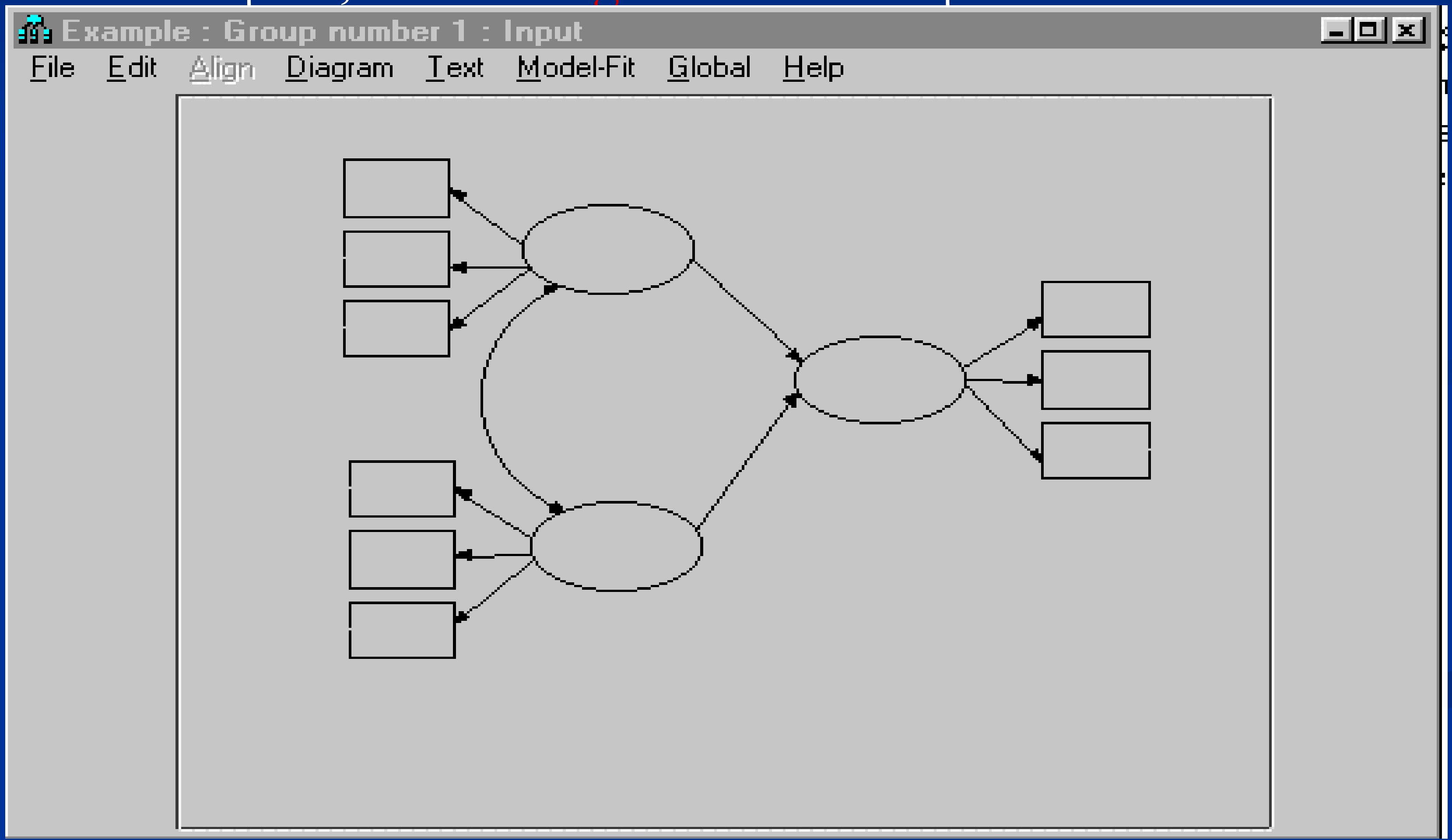
- In Amos Graphics, a model can be specified by drawing a diagram on the screen



1. To draw an observed variable, click "Diagram" on the top menu, and click "Draw Observed." Move the cursor to the place where you want to place an observed variable and click your mouse. Drag the box in order to adjust the size of the box. You can also use  in the tool box to draw observed variables.
2. Unobserved variables can be drawn similarly. Click "Diagram" and "Draw Unobserved." Unobserved variables are shown as circles. You may also use  in the toolbox to draw unobserved variables.

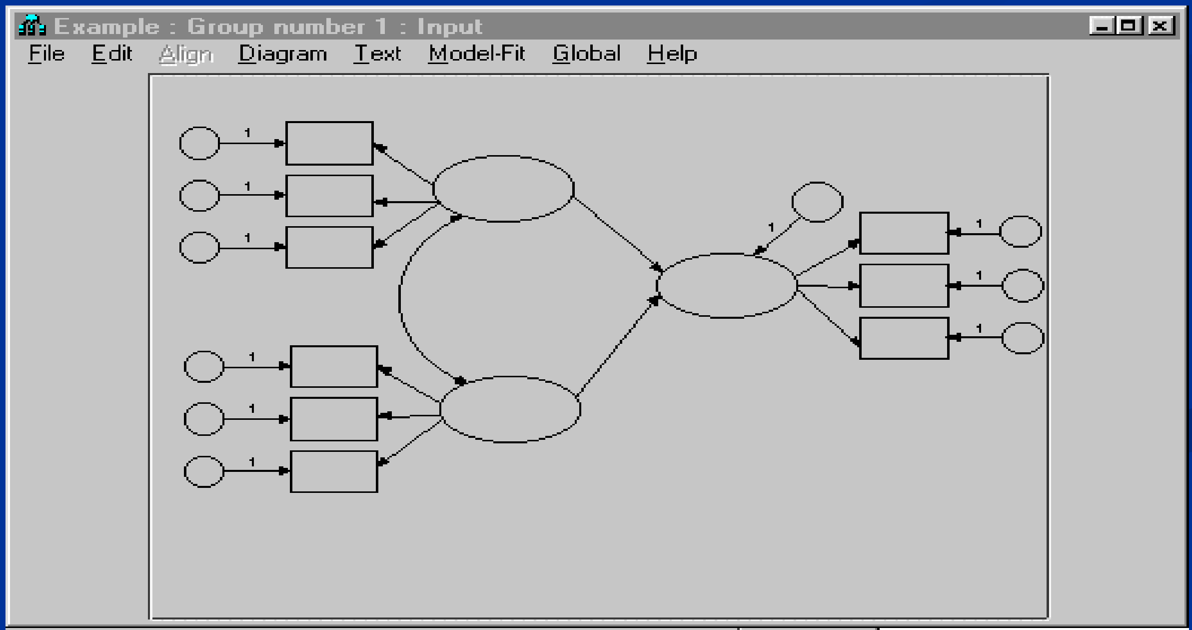
Drawing in AMOS

- To draw a path, Click "Diagram" on the top menu and click



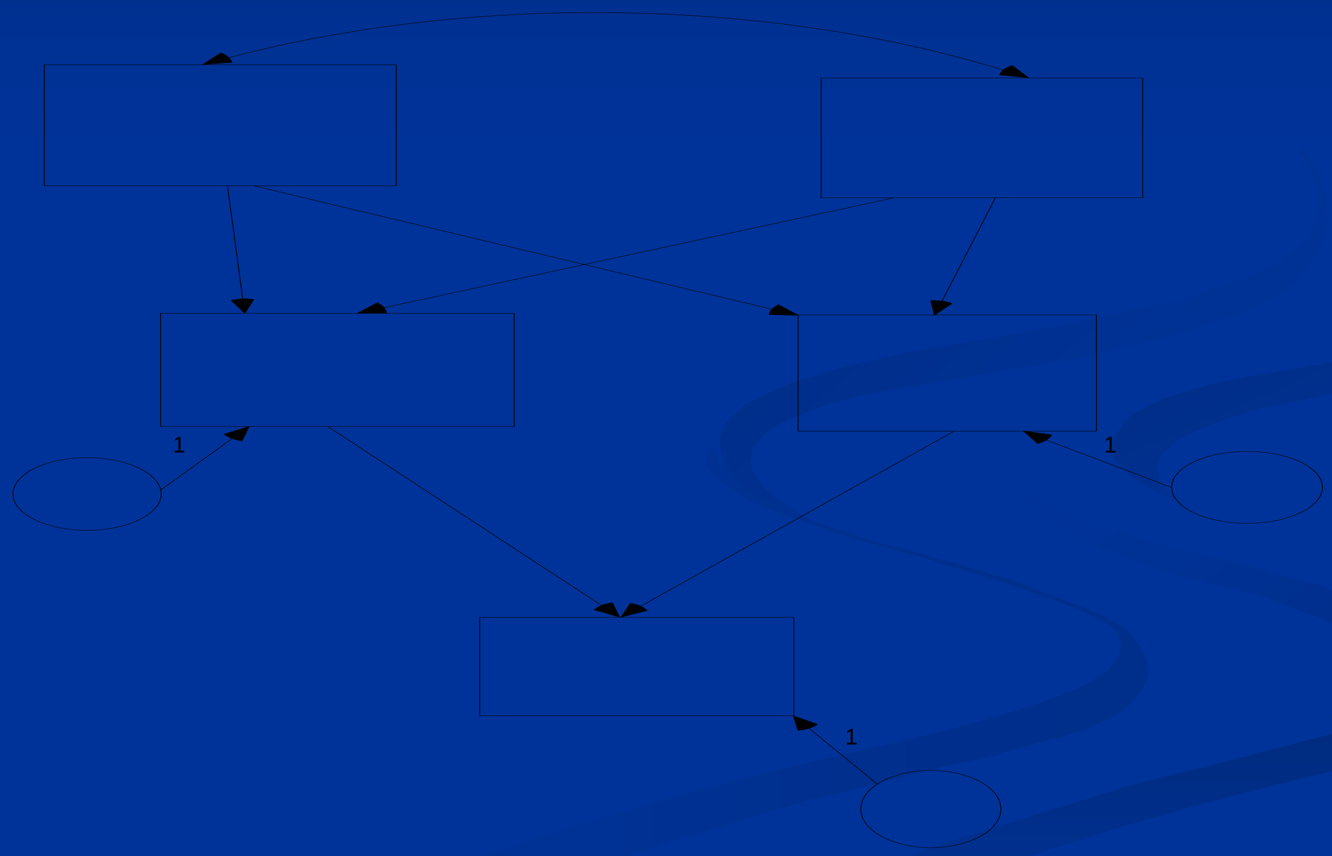
Drawing in AMOS

- To draw Error Term to the observed and unobserved variables.
- Use “Unique Variable” button in the Tool Box. Click ar  then click a box or a circle to which you want to add errors or a unique variables.(When you use "Unique Variable" button, the path coefficient will be automatically constrained to 1.)



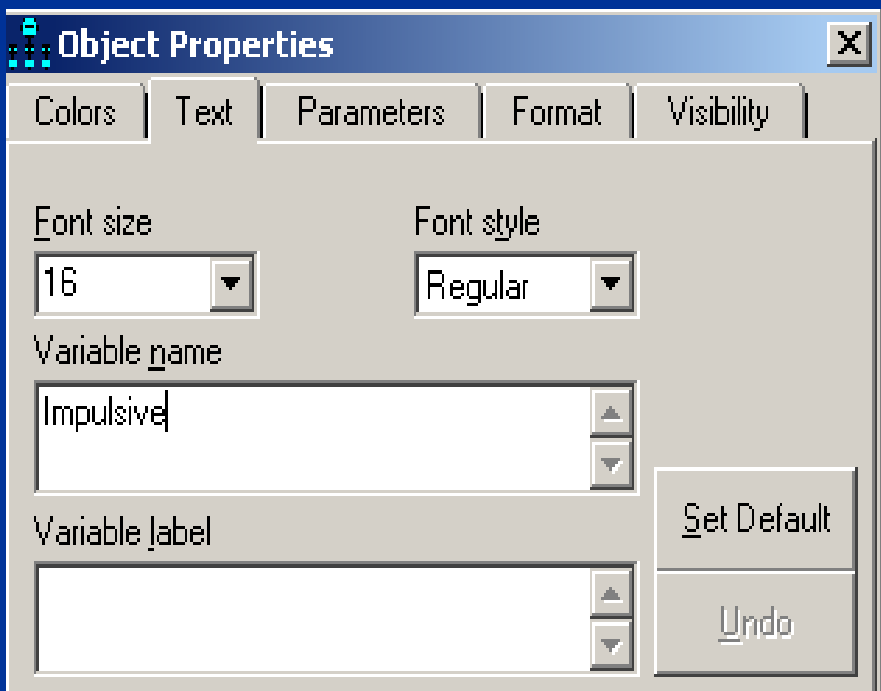
Drawing in AMOS

- Let us draw:



Naming the variables in AMOS

- double click on the objects in the path diagram. The Object Properties dialog box appears.

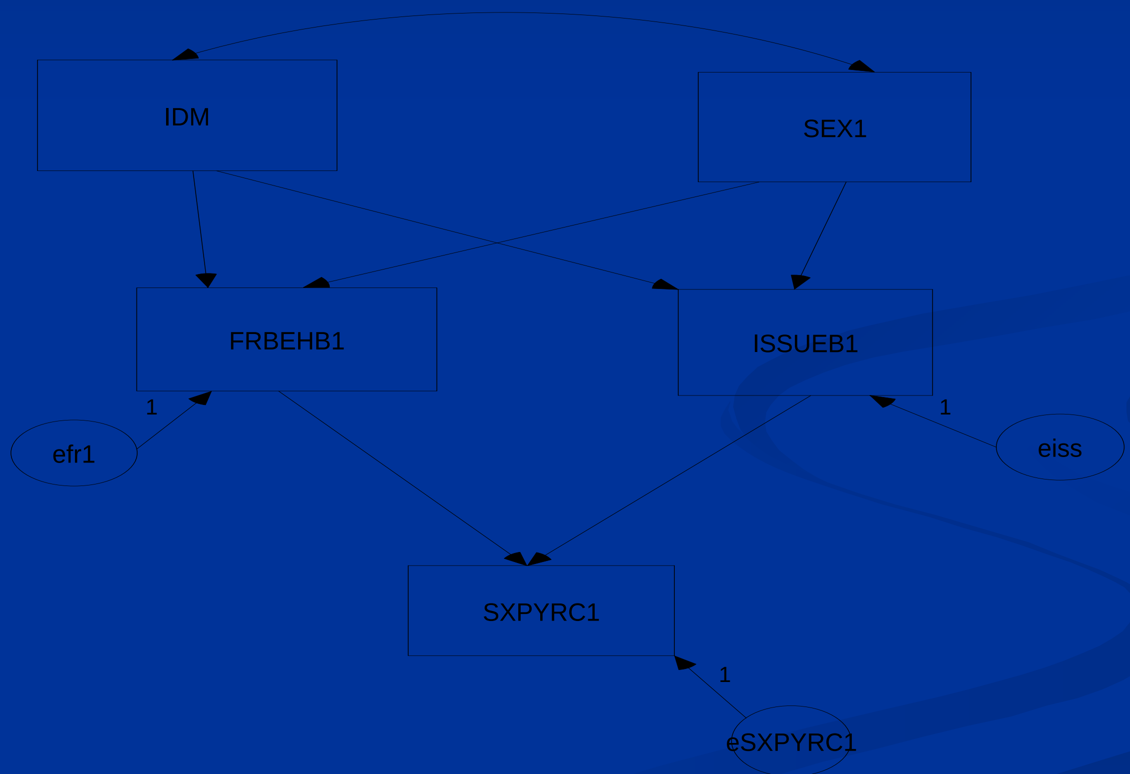


- OR

Click on the Text tab and enter the name of the variable in the Variable name field:

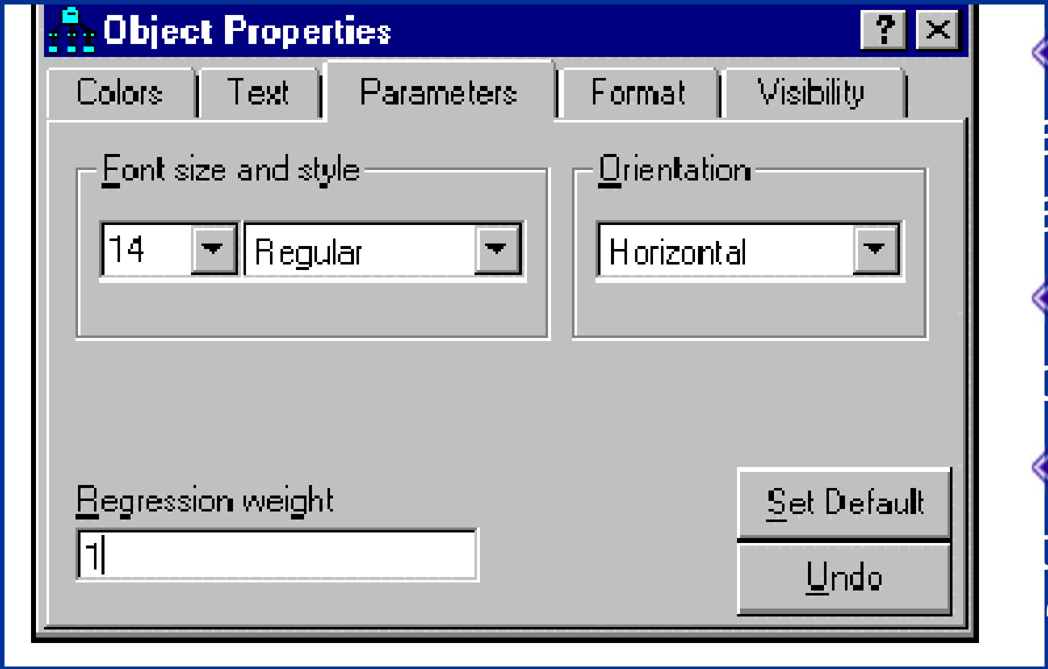
Naming the variables in AMOS

- Example: Name the variables



Constraining a parameter in AMOS

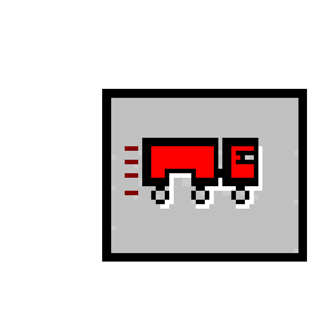
- The scale of the latent variable or variance of the latent variable has to be fixed to 1.



- Double click on the arrow between *EXPYA2* and *SXPYRA2*.
- The Object Properties dialog appears.
- Click on the Parameters tab and enter the value "*1*" in the Regression weight field:

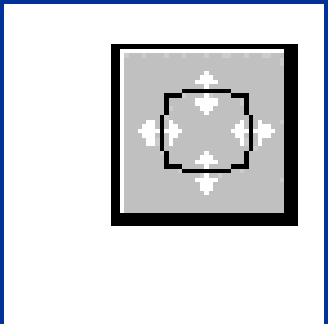
Improving the appearance of the path diagram

- You can change the appearance of your path diagram by moving objects around



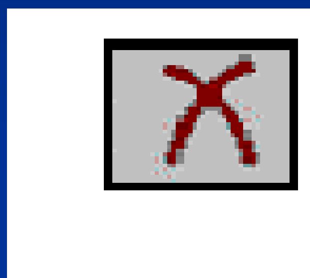
- To move an object, click on the Move icon on the toolbar. You will notice that the picture of a little moving truck appears below your mouse pointer when you move into the drawing area. This lets you know the Move function is active.
- Then click and hold down your left mouse button on the object you wish to move. With the mouse button still depressed, move the object to where you want it, and let go of your mouse button. Amos Graphics will automatically redraw all connecting arrows.

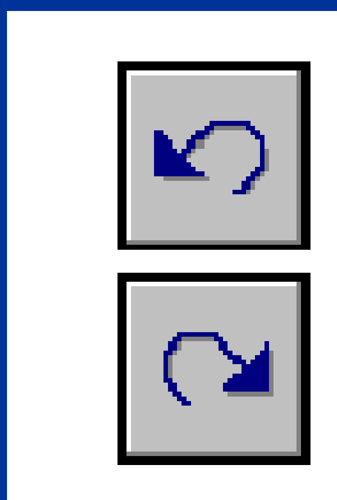
Improving the appearance of the path diagram



- To change the size and shape of an object, first press the Change the shape of objects icon on the toolbar.
- You will notice that the word "*shape*" appears under the mouse pointer to let you know the Shape function is active.
- Click and hold down your left mouse button on the object you wish to re-shape. Change the shape of the object to your liking and release the mouse button.
- Change the shape of objects also works on two-headed arrows. Follow the same procedure to change the direction or arc of any double-headed arrow.

Improving the appearance of the path diagram



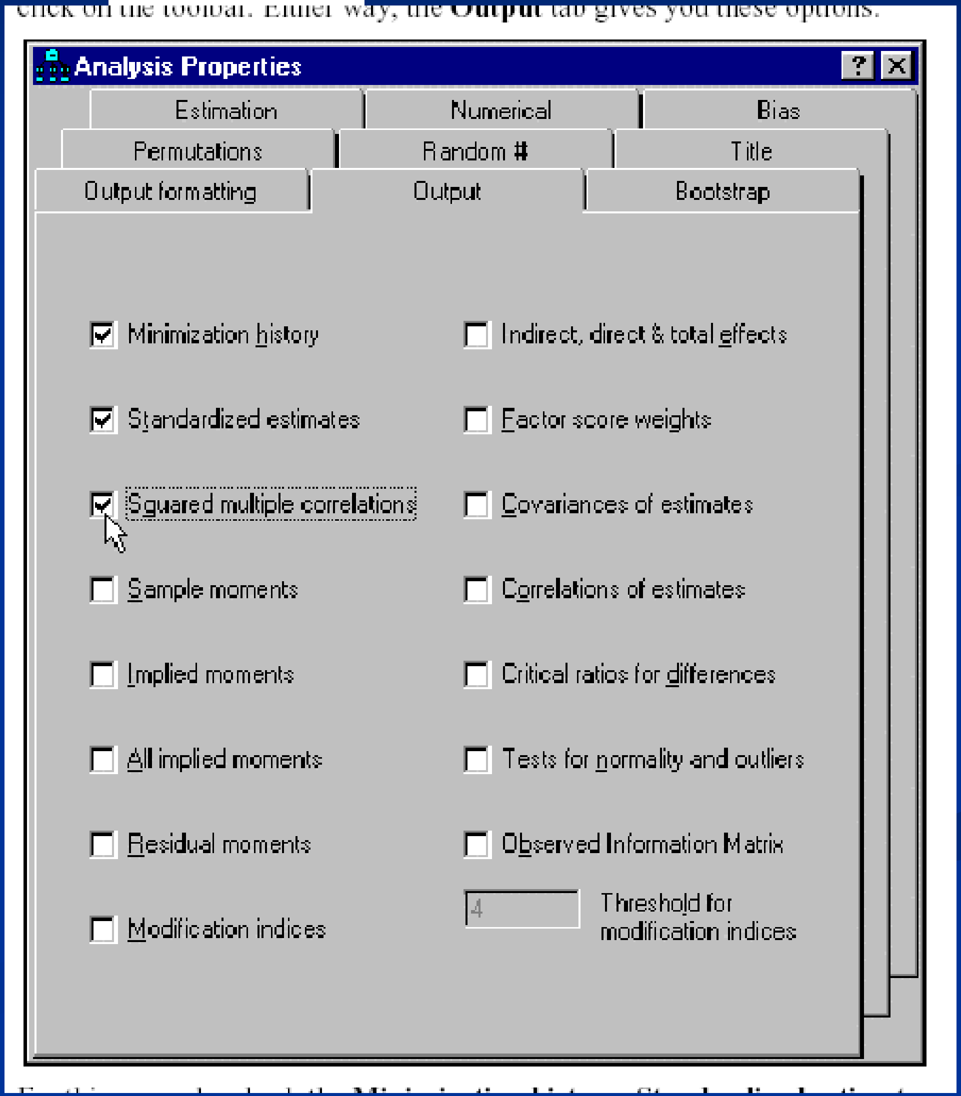


- If you *make a mistake*, there are always three icons on the toolbar to quickly bail you out: the Erase and Undo functions.
- To *erase an object*, simply click on the Erase icon and then click on the object you wish to erase.
- To *undo your last drawing activity*, click on the Undo icon and your last activity disappears.
- Each time you click Undo, your previous activity will be removed.
- *If you change your mind*, click on Redo to restore a change.

Performing
analysis in A

- View/Set → Analysis Properties and click on the Output tab.

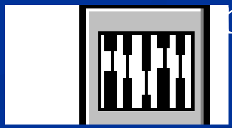
- There is also an Analysis Properties icon you can click on the toolbar. Either way, the Output tab gives you the following options:



Performing the analysis in AMOS

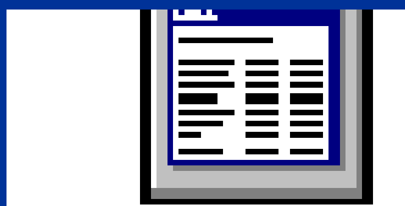
- For our example, check the Minimization history, Standardized estimates, and Squared multiple correlations boxes. (*We are doing this because these are so commonly used in analysis*).

- To run AMOS, click on the Calculate estimates icon on the toolbar.



  - AMOS will want to save this problem to a file.
  - if you have given it no filename, the Save As dialog box will appear. Give the problem a file name; let us say, **`tutorial1`**:

Results



- When AMOS has completed the calculations, you have two options for viewing the output:
  - *text output*,
  - *graphics output*.
- For text output, click the View Text (or F10) icon on the toolbar.
- Here is a portion of the text output for this problem:

Results for Condom Use Model(see handout)

The model is recursive. Sample size = 893

Chi-square=12.88 Degrees of Freedom =3

Maximum Likelihood Estimates

			Estimate	S.E.	C.R.	P
FRBEHB1	<---	SEX1	-.28	.09	-2.98	.00
ISSUEB1	<---	SEX1	.30	.08	3.79	***
FRBEHB1	<---	IDM	-.38	.11	-3.29	***
ISSUEB1	<---	IDM	-.57	.10	-5.94	***
SXPYRC1	<---	ISSUEB1	.16	.05	3.42	***
SXPYRC1	<---	FRBEHB1	.49	.04	12.21	***

Standardized Regression Weights: (Group number 1 - Default model)

			Estimate
FRBEHB1	<---	SEX1	-.10
ISSUEB1	<---	SEX1	.12
FRBEHB1	<---	IDM	-.11
ISSUEB1	<---	IDM	-.19
SXPYRC1	<---	ISSUEB1	.11
SXPYRC1	<---	FRBEHB1	.38

Results for Condom Use Model

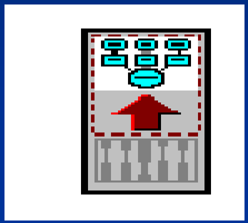
Covariances: (Group number 1 - Default model)

			Estimate	S.E.	C.R.	P	Label
SEX1	<-->	IDM	-.02	.01	-2.48	.01	

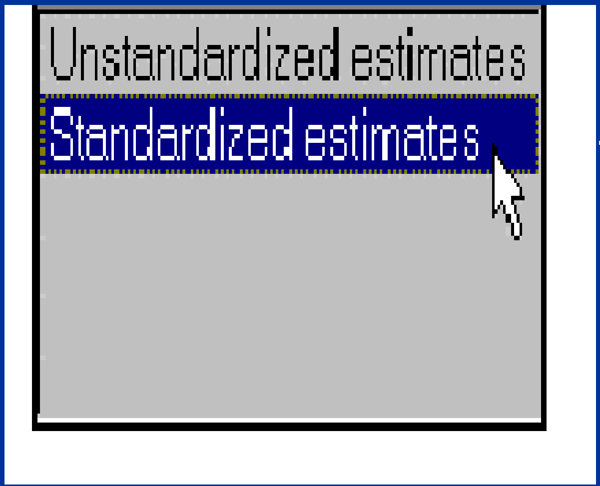
Correlations: (Group number 1 - Default model)

			Estimate
SEX1	<-->	IDM	-.08

Viewing the graphics output in AMOS



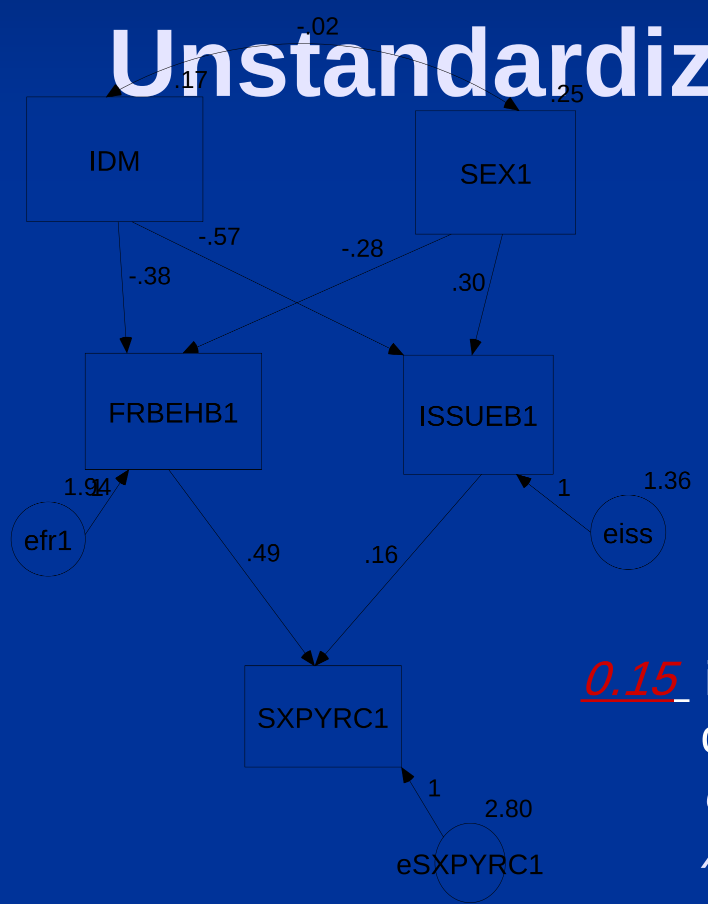
• To view the graphics output, click the View output icon next to the drawing area.



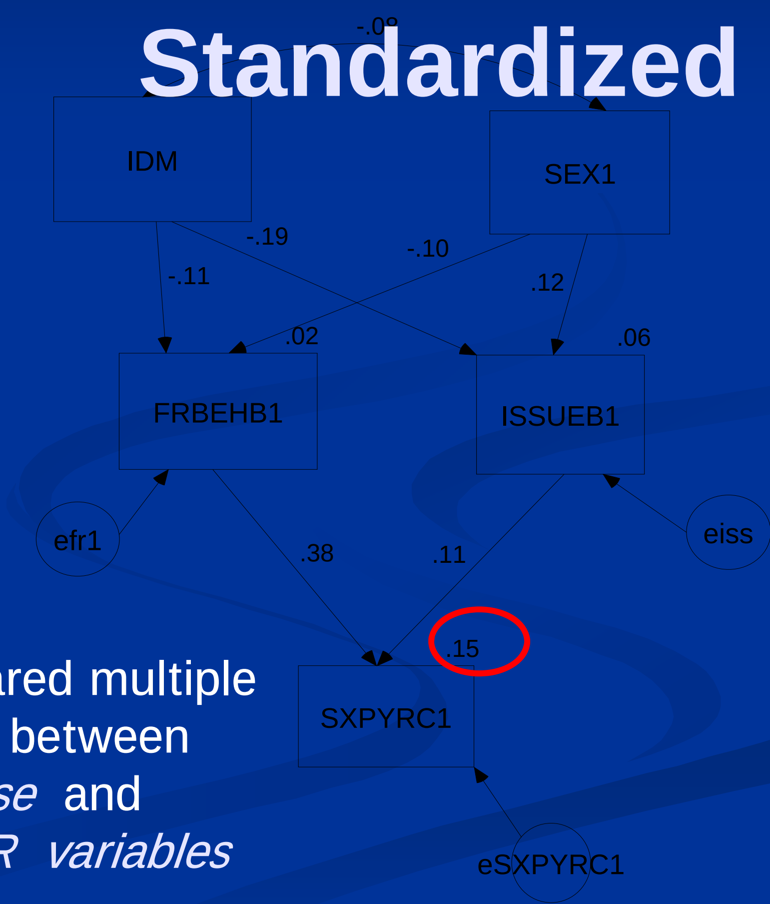
• Chose to view either *unstandardized* or (if you selected this option) *standardized* estimates by click one or the other in the Parameter Formats panel next to your drawing area:

Viewing the graphics output in AMOS

Unstandardized



Standardized



0.15 is the squared multiple correlation between *Condom use* and *ALL OTHER variables*

How to read the Output in AMOS

See the handout_1

Section 7

Putting it All Together

Section 6

Model Testing and Fit Indices,
Statistical Power

Model Specification

- Use theory to determine variables and relationships to test
- Fix, free, and constrain parameters as appropriate

Estimation Methods

- Maximum Likelihood—estimates maximize the likelihood that the data (observed covariances) were drawn from this population. Most forms are simultaneous. The fitting function is related to discrepancies between observed covariances and those predicted by the model. Typically iterative, deriving an initial solution then improves is through various calculations.
- Generalized and Unweighted Least Squares-- based on least squares criterion (rather than discrepancy function) but estimate all parameters simultaneously.
- 2-Stage and 3-Stage Least Squares—can be used to estimate non-recursive models, but estimate only one equation at a time. Applies multiple regression in two stages, replacing problematic variables (those correlated to disturbances) with a newly created predictor (instrumental variable that has direct effect on problematic variable but not on the endogenous variable).

Does the model "fit"?

- Model fit = sample data are consistent with the implied model
- The smaller the discrepancy between the implied model and the sample data, the better the fit.
- Model fit is Achilles' heel of SEM
  - Many fit indexes
  - None are fallible (though some are better than others)

Measures of Model Fit

- χ^2 = N-1 * minimization criterion. Just-identified model has = 0, no df. As chi-square increases, fit becomes worse. Badness of fit index. Tests difference in fit between given overidentified model and just-identified version of it.
- RMSEA—parsimony adjusted index to correct for model complexity. Approximates non-central chi-square distribution, which does not require a true null hypothesis, i.e., not a perfect model. Noncentrality parameter assesses the degree of falseness of the null hypothesis. Badness of fit index, with 0 best and higher values worse. Amount of error of approximation per model df. RMSEA $\leq$ .05 close fit, .05-.08 reasonable, $\geq$ .10 poor fit
- CFI—Assess fit of model compared to baseline model, typically independence or null model, which assumes zero population covariances among the observed variables
- AIC—used to select among nonhierarhical models

Model Fit

χ^2 Goodness of Fit test

- Historically used
- Desire a nonsignificant *p*-value, i.e., $p > .05$
- Adversely affected by sample size
  (N-1)*minimization function
- Badness of fit index
- Tests difference in fit between overidentified model and its just-identified version.
- Mixed opinions on its value in reporting.

Model Fit

CFI

- Fit determined by comparing implied model to a baseline model which assumes zero population covariances among the observed variables
- Initially, Bentler CFI > .90
- Hu & Bentler (1998, 1999) CFI > .95.

Model Fit

RMSEA

- Root Mean Squared Error of Approximation
- Adjusts fit index to correct for model complexity
- Based on noncentrality parameter which assesses the degree of falseness of the null hypothesis.
- Badness of fit index; 0 best & higher values worse.
- Amount of error of approximation per model df.
- RMSEA $\leq$ .05 close fit
- .05-.08 reasonable and $\geq$ .10 poor fit
- ALWAYS REPORT CONFIDENCE INTERVAL!

Model Fit

- Many other fit indexes
- Ideally
  - Nonsignificant χ^2 Goodness of Fit test
  - CFI > .95
  - RMSEA > .08
- IF model fits, then look at paths

Model Fit & Respecification

What if the model does NOT fit?

- Model trimming and building
  - LaGrange Multiplier test (add parameters)
  - Wald test (drop parameters)
- Empirical vs. theoretical respecification
  - What justification do you have to respecify?
- Consider equivalent models

Model Respecification

- Model trimming and building
- Empirical vs. theoretical respecification
- Consider equivalent models

Comparison of Models

- Hierarchical Models:
  - Difference of χ^2 test
- Non-hierarchical Models:
  - Compare model fit indices

Sample Size Guidelines

- Small (under 100), Medium (100-200), Large (200+) [try for medium, large better]
- Models with 1-2 df may require samples of thousands for model-level power of .8.
- When df=10 may only need n of 300-400 for model level power of .8.
- When df > 20 may only need n of 200 for power of .8
- 20:1 is ideal ratio for # cases/# free parameters, 10:1 is ok, less than 5:1 is almost certainly problematic
- For regression, $N \geq 50 + 8m$ for overall R^2, with m = # IVs and $N \geq 104 + m$ for individual predictors

Statistical Power

- Use power analysis tables from Cohen to assess power of specific detecting path coefficient.
- Saris & Satorra: use χ^2 difference test using predicted covariance matrix compared to one with that path = 0
- McCallum et al. (1996) based on RMSEA and chi-square distribution for close fit, not close fit and exact fit
- Small number of computer programs that calculate power for SEM at this point

Power Analysis for testing DATA-MODEL fit

- H_0: $\varepsilon_0 \geq 0.05$

  The Null hypothesis: The data-model fit is unacceptable

- H_1: $\varepsilon_1 < 0.05$

  The Alternative hypothesis: The data-model fit is acceptable

If RMSEA from the model fit is less than 0.05, then the null hypothesis containing unacceptable population data-model fit is rejected

Post Hoc Power Analysis for testing Data-Model fit

- If ε_1 is close to 0 → Power increases
- If N (sample size) increases → Power increases
- If df (degree of freedom) increases→ Power increases

Post Hoc Power Analysis for testing Data-Model fit

Examples Using Appendix B calculate power

for ε_1 =0.02, df=55, N=400→ Power ?

for ε_1 =0.04, df=30, N=400→ Power ?

Section 7:
Confirmatory Factor Analysis

Factor Analysis

Single Measure in Path Analysis

- Measurement error is higher

Multiple Measures in Factor Analysis correspond to some type of HYPOTHETICAL CONSTRUCT

- Reduce the overall effect of measurement error

Latent Construct

- Theory guides through the scale development process (DeVellis,1991; Jackson, 1970)
- Unidimensional vs Multidimensional constuct
- Reliability and Validity of construct

Reliability - consistency, precision, repeatability

Reliability concerns with RANDOM ERROR

Types of reliability:

- test-retest
- alternate form
- interrater
- split-half and internal consistency

Validity of construct

4 types of validity

- content
- criterion-related
- convergent and discriminant
- construct

Factor analysis

- Indicators: **continuous**
- Measurement error are independent of each other and of the factors
- All associations between the factors are unanalyzed

Two Classes of Factor Analysis

- Exploratory Factor Analysis
  - Exploring possible factors
  - Factor analysis you're probably used to
- Confirmatory Factor Analysis
  - Testing possible models of factor structure
  - Using previous findings

Identification of CFA

- Can estimate $v*(v+1)/2$ of parameters
- Necessary
  - # of free parameters <= # of observations
  - Every latent variable should be **scaled**

Additional: fix the unstandardized residual path of the error to **1.** (assign a scale of the unique variance of its indicator)

Scaling factor: constrain one of the factor loadings to 1

(that variables called – reference variable, the factor has a scale related to the explained variance of the reference variable)

OR

fix factor variance to a constant (ex. 1), so all factor loadings are free parameters

Both methods of scaling result in the same overall fit of the model

Identification of CFA

- Sufficient :
  - At least three (3) indicators per factor to make the model identified
  - Two-indicator rule – prone to estimation problems (esp. with small sample size)

Interpretation of the estimates

- **Unstandardized solution**
  - Factor loadings =unstandardized regression coefficient
  - Unanalyzed association between factors or errors= covariances
- **Standardized solution**
  - Unanalyzed association between factors or errors= correlations
  - Factor loadings =standardized regression coefficient

    (structure coefficient)
  - The square of the factor loadings = the proportion of the explained (common) indicator variance, R^2(squared multiple correlation)

Problems in estimation of CFA

- Heywood cases – negative variance estimated or correlations > 1.
- Ratio of the sample size to the free parameters – 10:1 (better 20:1)
- Nonnormality – affects ML estimation

Suggestions by March and Hau(1999)when sample size is small:

  - indicators with high standardized loadings(>0.6)
  - constrain the factor loadings

Testing CFA models

- Test for a single factor with the theory or not
- If reject H_0 of good fit - try two-factor model…
- Since one-factor model is restricted version of the two - factor model , then compare one-factor model to two-factor model using Chi-square test . If the Chi-square is significant – then the 2-factor model is better than 1-factor model.
- Check R^2 of the unexplained variance of the indicators.

Respecification of CFA

IF

- lower factor loadings of the indicator (standardized<=0.2)
- High loading on more than one factor
- High correlation of the residuals
- High factor correlation

THEN

- Specify that indicator on a different factor
- Allow to load on one more than one factor

(multidimensional vs unidimensional)

- Allow error measurements to covary
- Too many factors specified

Other tests

- Indicators:
  - congeneric – measure the same construct

    if model fits , then

    -tau-equivalent – constrain all unstandardized loadings to 1

    if model fit, then

    - parallelism – equality of error variances

→ All these can be tested by χ^2 difference test

Nonnormal distributions

- Normalize with transformations
- Use **corrected normal theory method,** e.g. use robust standard errors and corrected test statistics, (Satorra-Bentler statistics)
- Use Asymptotic distribution free or arbitrary distribution function (ADF) - no distribution assumption - Need large sample
- Use elliptical distribution theory – need only symmetric distribution
- Mean-adjusted weighted least squares (MLSW) and variance-adjusted weighted least square (VLSW) - MPLUS with categorical indicators
- Use normal theory with nonparametric bootstrapping

Remedies to nonnormality

- Use a parcel which is a linear composite of the discrete scores, as continuous indicators
- Use parceling ,when underlying factor is unidimentional.

Section 8:

Putting it All Together: Structural Regression Models

Testing Models with Structural and Measurement Components

- Identification Issues
  - For the structural portion of SR model to be identified, its measurement portion must be identified.
  - Use the two-step rule: Respecify the SR model as CFA with all possible unanalyzed associations among factors. Assess identificaiton.
  - View structural portion of the SR model and determine if it is recursive. If so, it is identified. If not, use order and rank conditions.

The 2-Step Approach

- Anderson & Gerbing's approach
  - Saturated model, theoretical model of interest
  - Next most likely constrained and unconstrained structural models
- Kline and others' 2-step approach:
  - Respecify SR as CFA. Then test various SR models.

The 4-Step Approach

- Factor Model
- Confirmatory Factor Model
- Anticipated Structural Equation Model
- More Constrained Structural Equation Model

Constraint Interaction

- When chi-square and parameter estimates differ depending on whether loading or variance is constrained.
- Test: If loadings have been constrained, change to a new constant. If variance constrained, fix to a constant other than 1.0. If chi-square value for modified model is not identical, constraint interaction is present. Scale based on substantive grounds.

Single Indicators in Partially Latent SR Models

Estimate proportion of variance of variable due to error (unique variance). Multiply by variance of measured variable.

Section 9

Multiple-Group Models,
a Word about Latent Growth Models,
Pitfalls, Critique and
Future Directions for SEM

Multiple-Group Models

- Main question addressed: do values of model parameters vary across groups?
- Another equivalent way of expressing this question: does group membership moderate the relations specified in the model?
- Is there an interaction between group membership and exogenous variables in effect on endogenous variables?

Cross-group equality constraints

- One model is fit for each group, with equal unstandardized coefficients for a set of parameters in the model
- This model can be compared to an unconstrained model in which all parameters are unconstrained to be equal between groups

Latent Growth Models

- Latent Growth Models in SEM are often structural regression models with mean structures

Mean Structures

- Means are estimated by regression of variables on a constant
- Parameters of a mean structure include means of exogenous variables and intercepts of endogenous variables.
- Predicted means of endogenous variables can be compared to observed means.

Principles of Mean Structures in SEM

- When a variable is regressed on a predictor and a constant, the unstandardized coefficient for the constant is the intercept.
- When a predictor is regressed on a constant, the undstandardized coefficient is the mean of the predictor.
- The mean of an endogenous variable is a function of three parameters: the intercept, the unstandardized path coefficient, and the mean of the exogenous variable.

Requirements for LGM within SEM

- continuous dependent variable measured on at least three different occasions
- scores that have the same units across time, can be said to measure the same construct at each assessment, and are not standardized
- data that are time structured, meaning that cases are all tested at the same intervals (not need be equal intervals)

Pitfalls--Specification

- Specifying the model after data collection
- Insufficient number of indicators. Kenny: "2 might be fine, 3 is better, 4 is best, more is gravy"
- Carefully consider directionality
- Forgetting about parsimony
- Adding disturbance or measurement errors without substantive justification

Pitfalls--Data

- Forgetting to look at missing data patterns
- Forgetting to look at distributions, outliers, or non-linearity of relationships
- Lack of independence among observations due to clustering of individuals

Pitfalls—Analysis/Respecification

- Using statistical results only and not theory to respecify a model
- Failure to consider constraint interactions and Heywood cases (illogical values for parameters)
- Use of correlation matrix rather than covariance matrix
- Failure to test measurement model first
- Failure to consider sample size vs. model complexity

Pitfalls--Interpretation

- Suggesting that "good fit" proves the model
- Not understanding the difference between good fit and high R^2
- Using standardized estimates in comparing multiple-group results
- Failure to consider equivalent or (nonequivalent) alternative models
- Naming fallacy
- Suggesting results prove causality

Critique

- The multiple/alternative models problem
- The belief that the "stronger" method and path diagram proves causality
- Use of SEM for model modification rather than for model testing. Instead:
  - Models should be modified before SEM is conducted or
  - Sample sizes should be large enough to modify the model with half of the sample and then cross-validate the new model with the other half

Future Directions

- Assessment of interactions
- Multiple-level models
- Curvilinear effects
- Dichotomous and ordinal variables

Final Thoughts

- SEM can be useful, especially to:
  - separate measurement error from structural relationships
  - assess models with multiple outcomes
  - assess moderating effects via multiple-sample analyses
  - consider bidirectional relationships
- But be careful. Sample size concerns, lots of model modification, concluding too much, and not considering alternative models are especially important pitfalls.

AMOS, Part 2

Modification of the Model

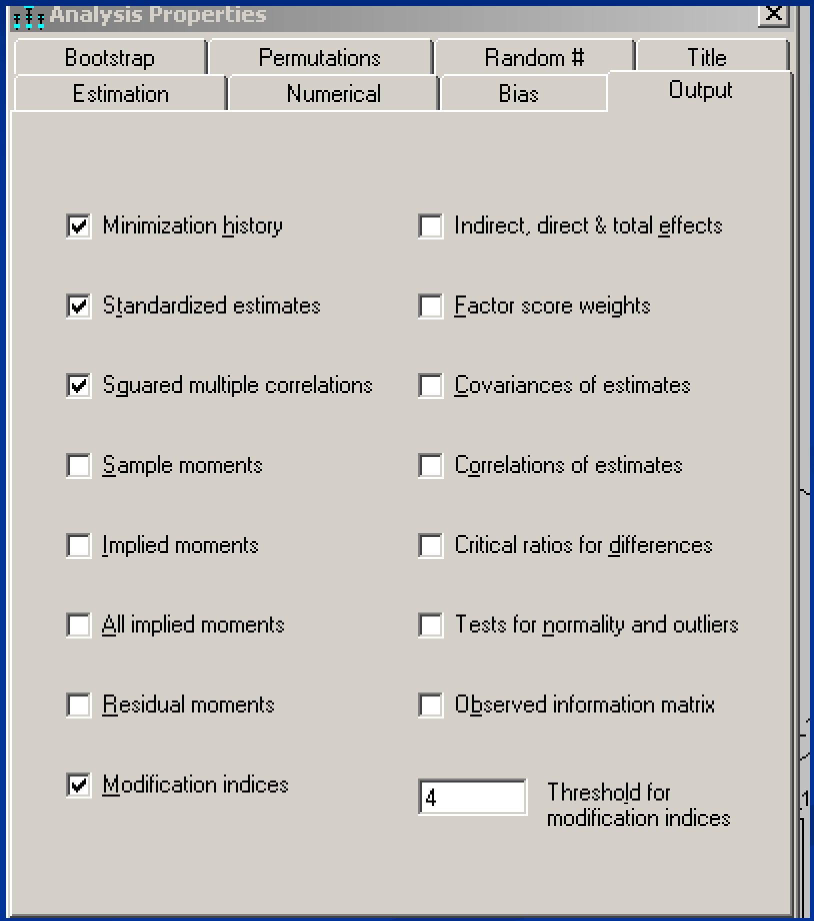
- Search for the better model

- Suggestions from: 1) theory

  2) modification indices using AMOS

Modifying the Model using AMOS

- View/Set → Analysis Properties and click on the Output tab.
- Then check the Modification indices option



Modifying the Model using AMOS

Modification Indices (Group number 1 - Default model)

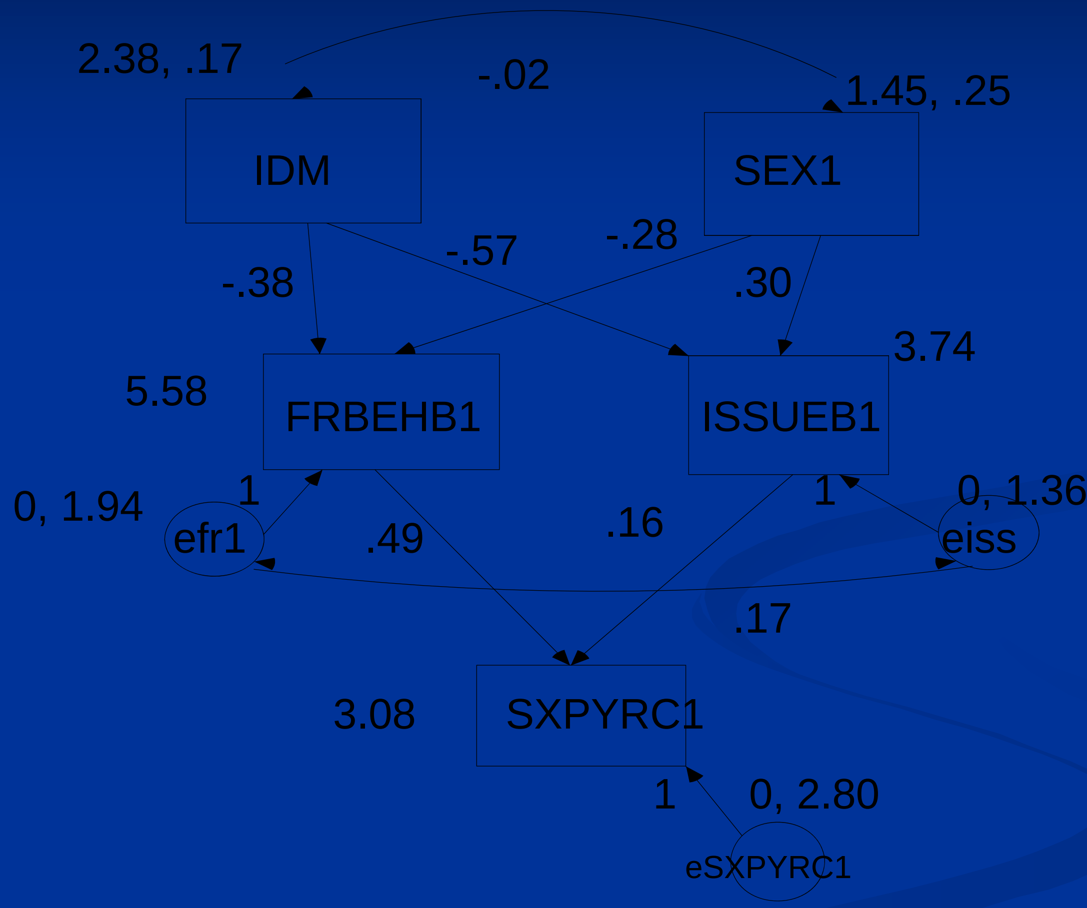
Covariances: (Group number 1 - Default model)

			M.I.	Par Change
eiss	<-->	efr1	9.909	.171

Parameter increase

Chi-square decrease

Modifying the Model using AMOS



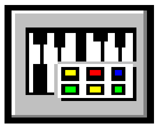
SEE Handout # 2 for the whole output

Examples using AMOS

- Condom Use Model with missing values
- Confirmatory Factor Analysis for Impulsive Decision Making construct
- Multiple group analysis
- How to deal with non-normal data

Missing data in AMOS

- Full Information Maximum Likelihood estimation

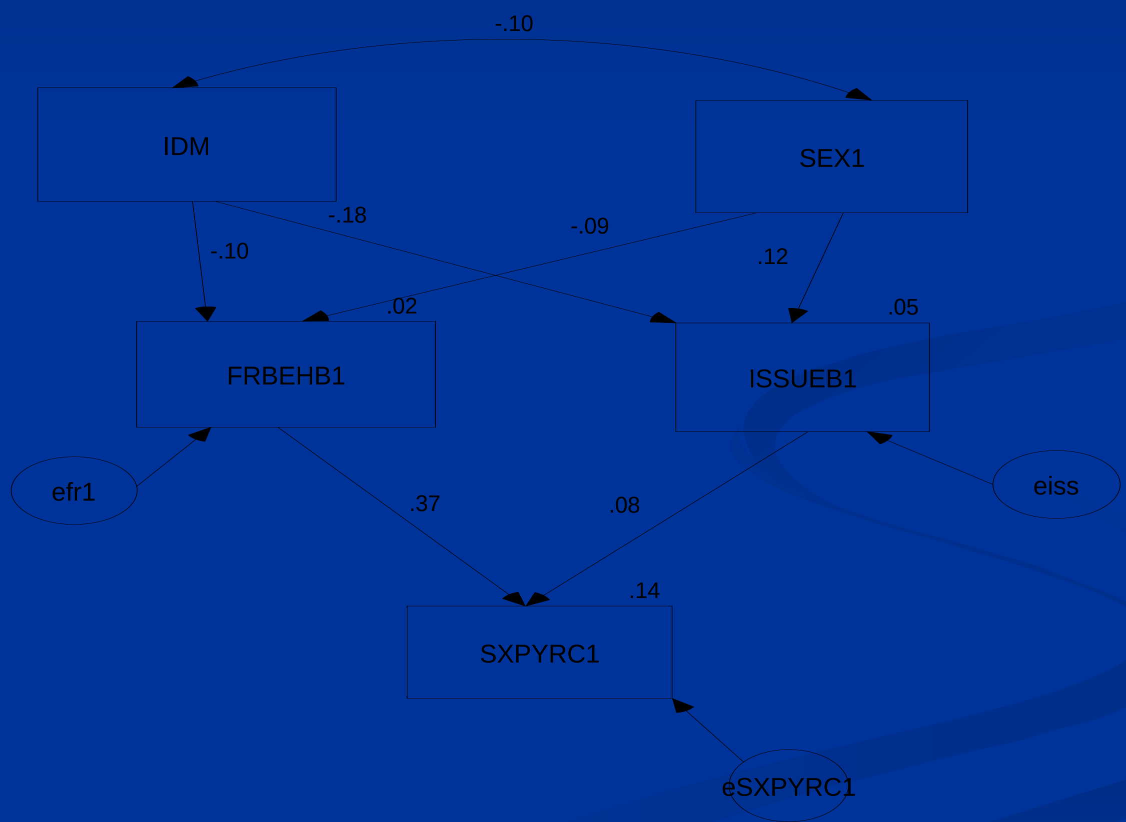


- View/Set -> Analysis Properties and click on the Estimation tab.
- Click on the button Estimate Means and Intercepts. This uses FIML estimation

- Recalculate the previous example with data "**`AMOS_data.sav`**" with some missing values

Missing data in AMOS

- The standardized graphical output.

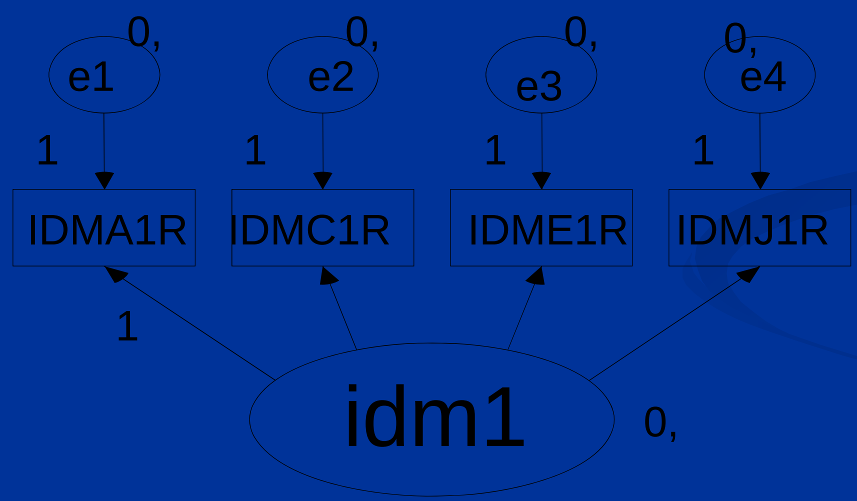


Missing data in AMOS

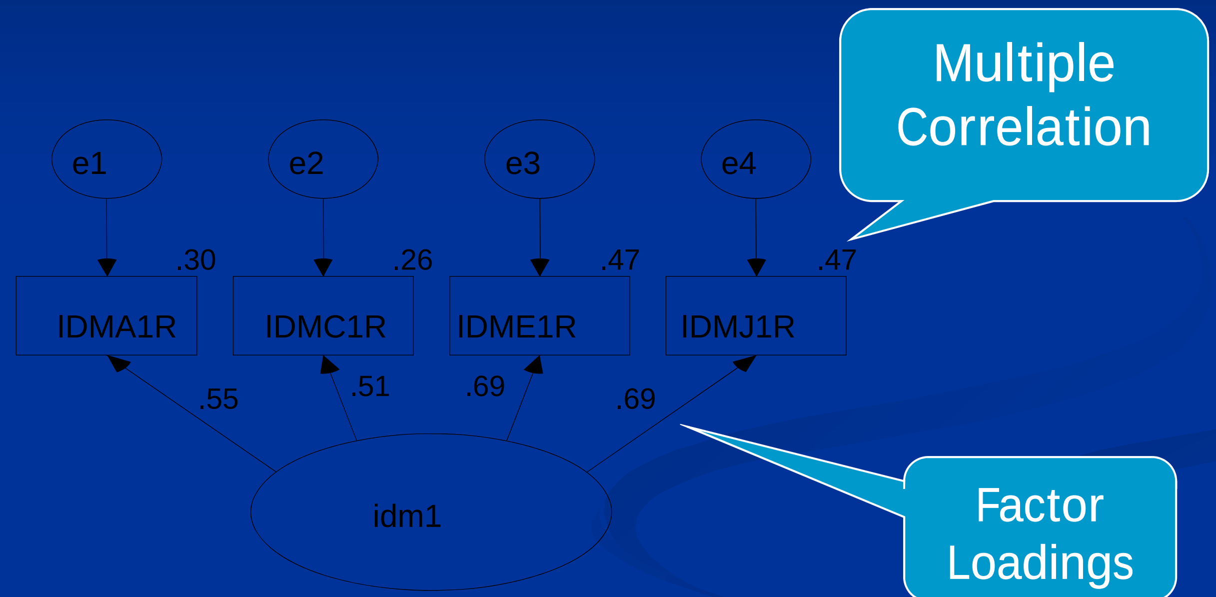
Example: see the handout #3

Confirmatory Factor Analysis with Impulsive Decision Making scale

- Need to fix either the variance of the IDM1 factor or one of the loadings to 1.



Confirmatory Factor Analysis with Impulsive Decision Making scale



Chi-square = 11.621 Degrees of freedom = 2, p=0.003
CFI=0.994, RMSEA=0.042

Confirmatory Factor Analysis with Impulsive Decision Making scale

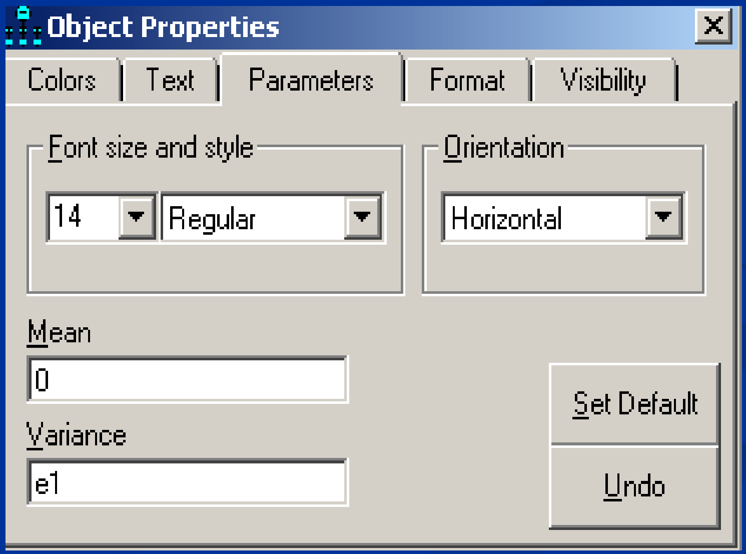
- What if want to *compare two NESTED models* for Impulsive Decision Making Model?

    1) error variances equal for all 4 measured variables

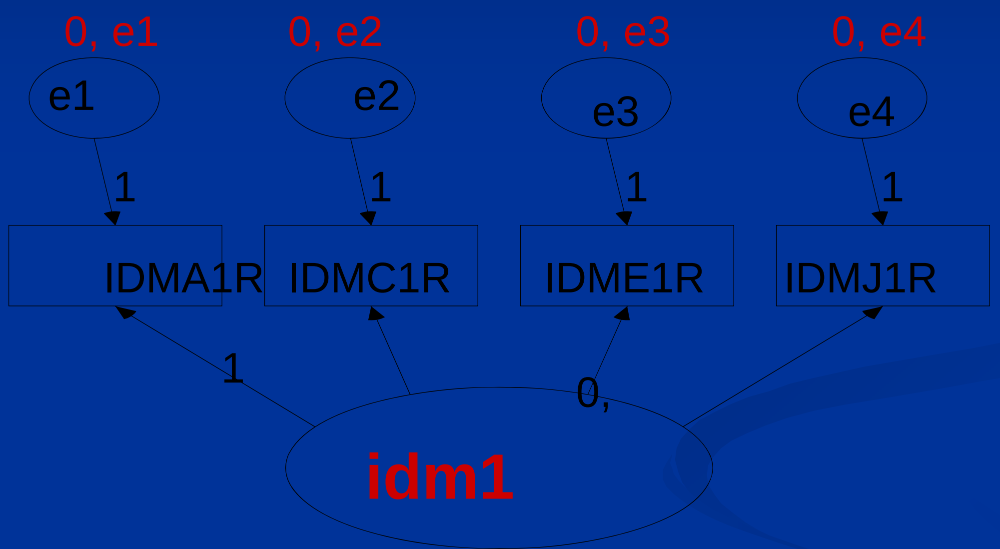
    2) error variances are different

Confirmatory Factor Analysis with Impulsive Decision Making scale: the error variances are the same

- Need to give names to the error variances, by double clicking on the error variance. The Object properties will appear, click on the Parameter and type the name for the error variance(e1, e2...) in the Variance box.

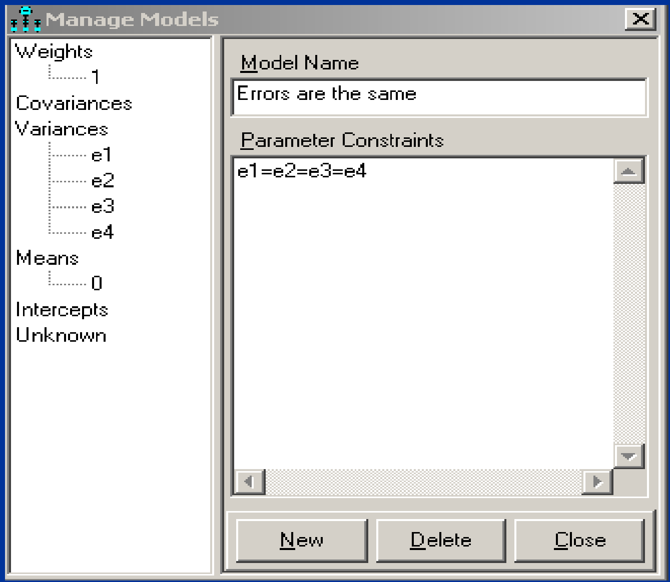


Confirmatory Factor Analysis with Impulsive Decision Making scale



Confirmatory Factor Analysis with Impulsive Decision Making scale: error variances are the same

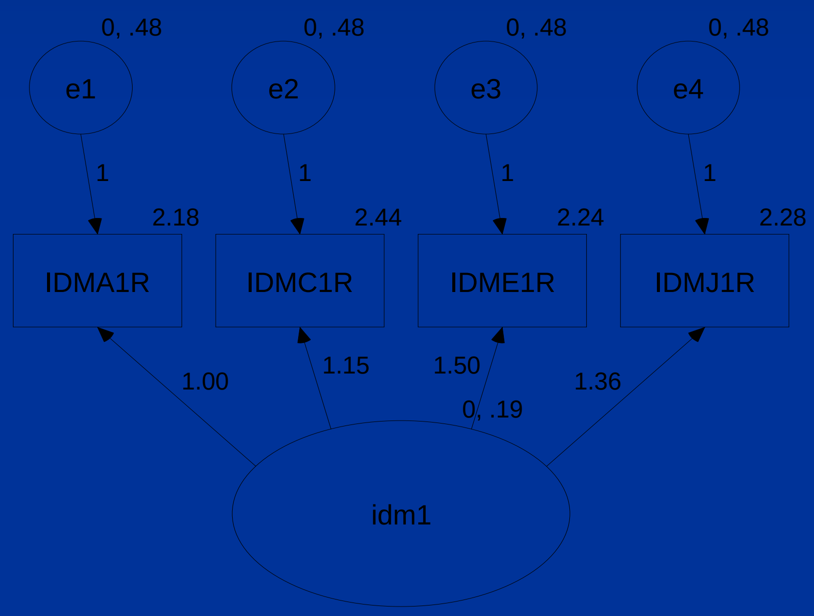
- Click MODEL FIT , then Manage Models
- In the Manage Models window, click on New.
- In the Parameter Constraints segment of the window type "**e1=e2=e3=e4**"



Now there are two nested models

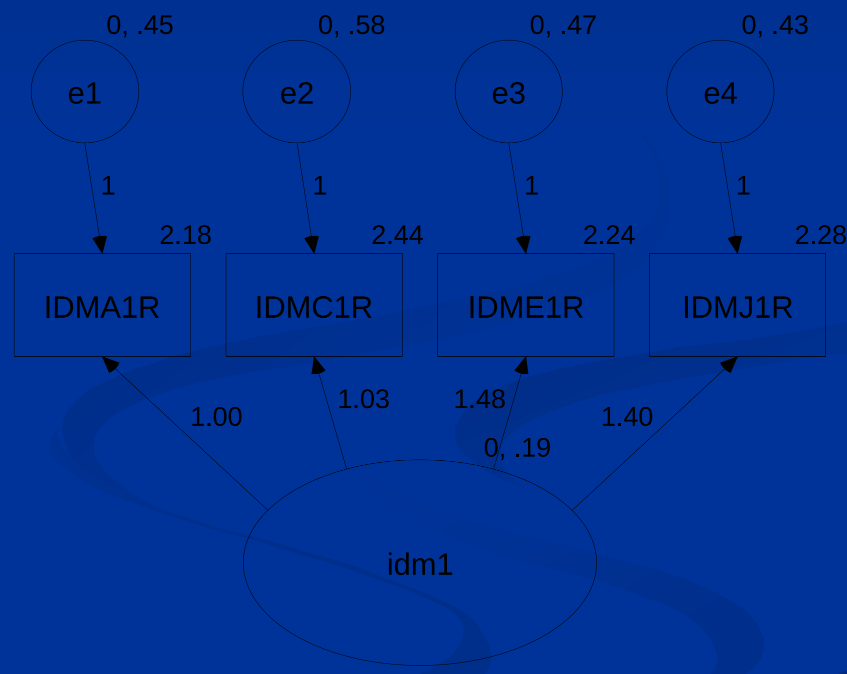
Confirmatory Factor Analysis with Impulsive Decision Making scale

error variances are the same



Chi-square = 56.826, df=5, p=0.000

error variances are different



Chi-square = 11.621, df=3, p=0.003

Confirmatory Factor Analysis with Impulsive Decision Making scale: error variances are the same

- Compare Nested Models using Chi-square difference test:

Model2(errors the same)
Chi-square = 56.826,
df=5, p=0.000

Model1 (errors are different)
Chi-square = 11.621,
df=3, p=0.003

- $\textit{Chi-square}_{difference} = 56.826 - 11.621 = 45.205$
- $df = 5 - 3 = 2$
- $\textit{Chi-square}_{critical\ value} = 5.99$ → Significant
- Model 2 with Equal error variances fits WORSE than Model 1

Confirmatory Factor Analysis with Impulsive Decision Making scale: error variances are the same

Nested Model Comparisons

Assuming model Error are free to be correct:

Model	DF	CMIN	P	NFI Delta-1	IFI Delta-2	RFI rho-1	TLI rho2
Errors are the same	3	45.205	.000	.026	.026	.032	.032

Multiple group analysis

- WHY: test the equality/invariance of the factor loadings for two separate groups
- HOW :

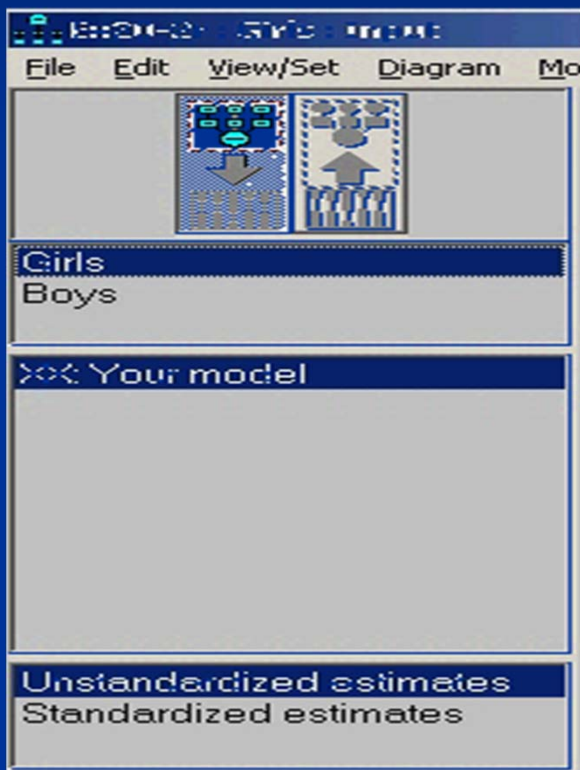
  1) test the model to *both groups separately* to check the entire model

  2) the same model by multiple group analysis

  Example: *Do Males and Females can be fitted to the same Condom USE model?*

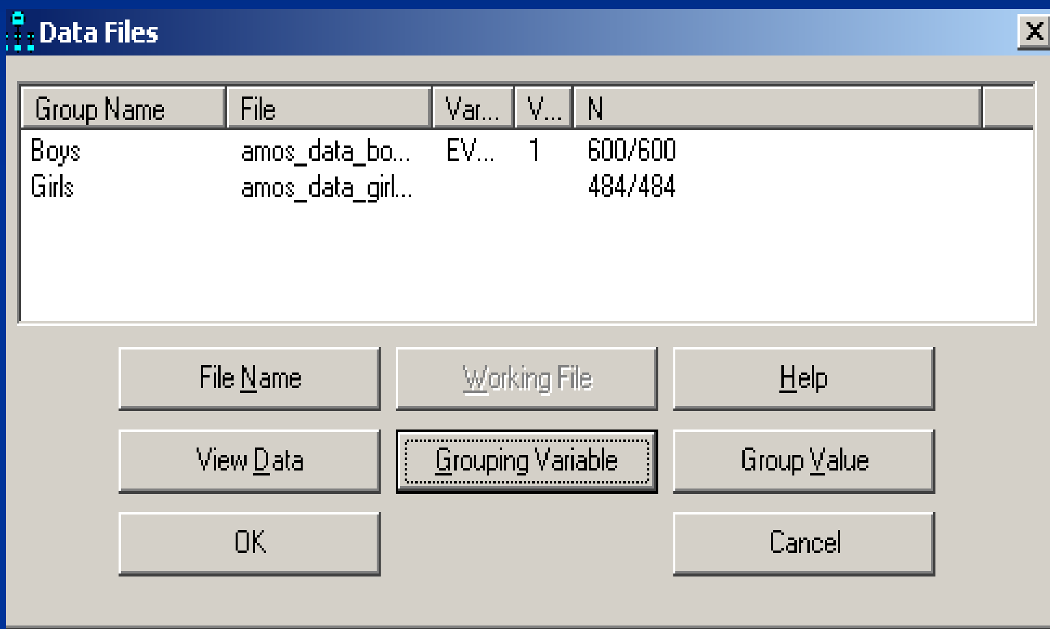
- Need to have 2 separate data files for each group.
  - **`data_boys`** and **`data_girls`**.

Multiple group analysis



- Select Manage Groups... from the Model Fit menu.
- Name the first group "Girls".
- Next, click on the New button to add a second group to the analysis.
- Name this group "Boys".
- AMOS 4.0 will allow you to consider up to 16 groups per analysis.
- Each newly created group is represented by its own path diagram

Multiple group analysis



- Select File->Data Files... to launch the Data Files dialog box.
- For each group, specify the relevant data file name.
- For this example, choose the **`data_girls`** SPSS database for the girls' group;
- choose the **`data_boys`** SPSS database for the boys' group.

Multiple group analysis

- The following models fit to both groups *(see handout)* :

Unconstrained – all parameters are different in each group

Measurement weights – regression loadings are the same in both groups

Measurement intercepts – the same intercepts for both groups

Structural weights – the same regression loadings between the latent var.

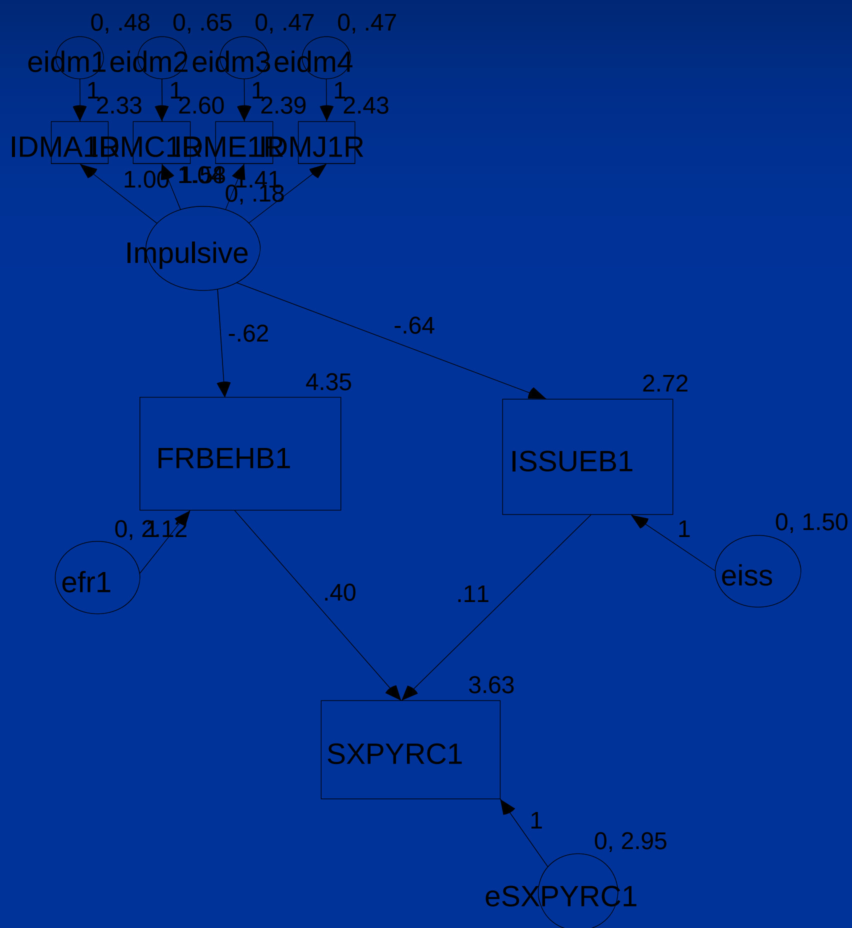
Structural intercepts – the same intercepts for the latent variables

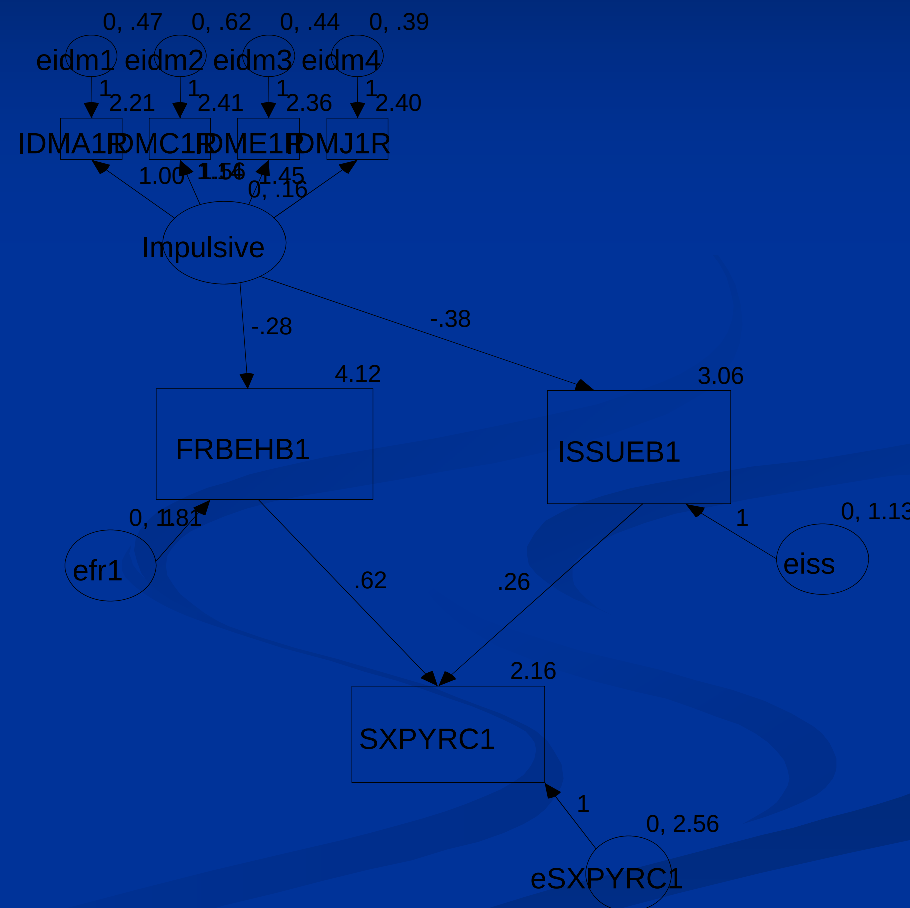
Structural covariates – the same variances/covariance for the latent var.

Structural residuals – the same disturbances

Measurement residuals – the same errors-THE MOST RESTRICTIVE MODEL

Example: Multiple group analysis for Condom use Model



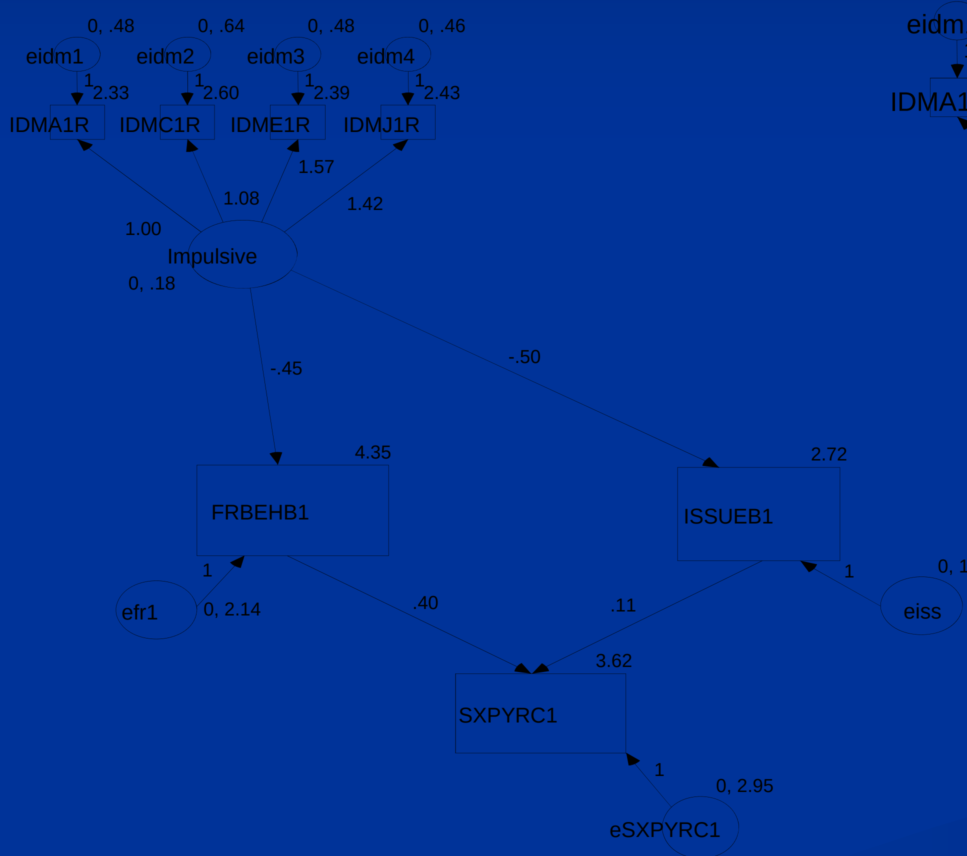


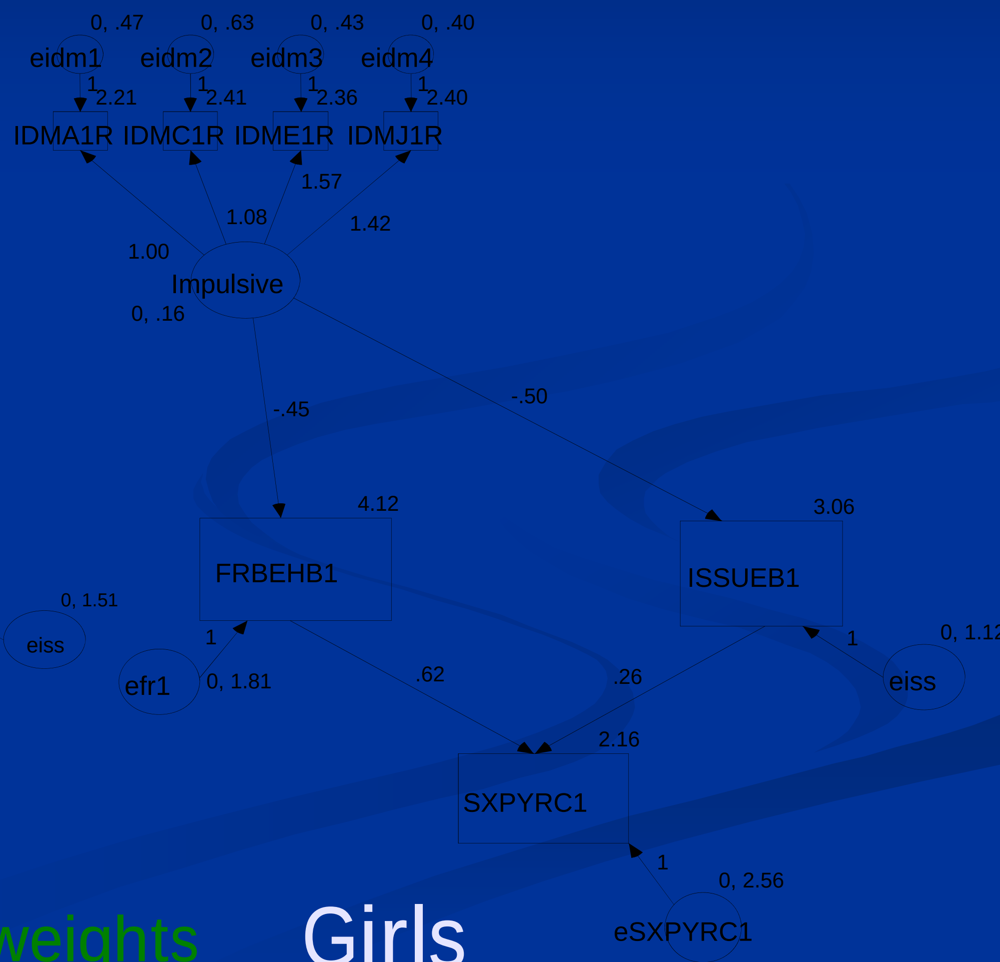
Boys

UNCONSTRAINED MODEL

Girls

Example: Multiple group analysis for Condom use Model





Boys Measurement weights Girls

Example: Multiple group analysis for Condom use Model

- *see handout*

- Since *Measurement Weights* model is nested within *Unconstrained* .

- Chi-square difference test computed to test the null hypothesis that the *regression weights for boys and girls* are the same. However, the *variances and covariance* are *different* across groups.

Example: Multiple group analysis for Condom use Model

Chi-square$_{diff}$ =68.901-65.119=2.282

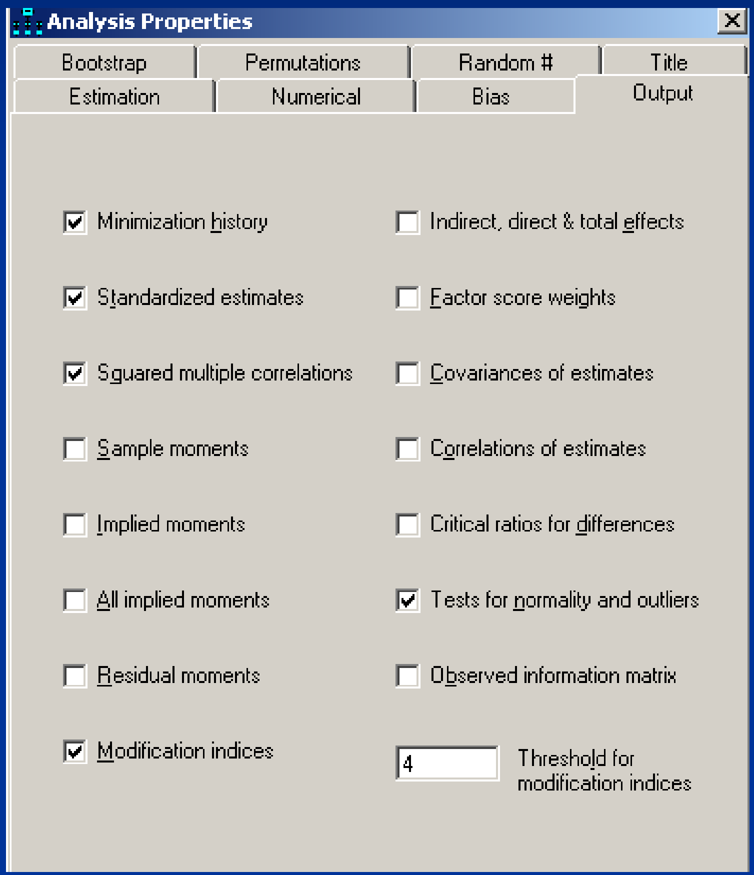
df=29-26=3 → *NOT SIGNIFICANT*

FIT of the Measurement Weights model is not significantly worse than Unconstrained

Handling non-normal data:

- Verify that your variables are not distributed joint multivariate normal
- Assess overall model fit using the Bollen-Stine corrected *p*-value
- Use the bootstrap to generate parameter estimates, standard errors of parameter estimates, and significance tests for individual parameters

Handling non-normal data: checking for normality



To verify that the data is not normal. Check the *Univariate SKEWNESS* and *KURTOSIS* for each variable .

- View/Set -> Analysis Properties

  and click on the Output tab.

- Click on the button Tests for normality and outliers

Handling non-normal data: checking for normality

Assessment of normality

Variable	min	max	skew	c.r.	kurtosis	c.r.
IDM	1.182	3.727	.381	4.649	.496	3.025
SEX1	1.000	2.000	.182	2.222	-1.967	-11.997
FRBEHB1	1.000	6.000	-.430	-5.245	-.778	-4.748
ISSUEB1	1.000	4.000	-.431	-5.259	-1.387	-8.462
SXPYRC1	2.000	7.000	-.937	-11.436	-.715	-4.360
Multivariate					**-3.443**	-6.149

Critical ratio of +/- 2 for skewness and kurtosis

→statistical significance of NON-NORMALLITY

Multivariate kurtosis >10 → Severe Non-normality

Upper critical values of chi-square distribution

Degree of freedom	Chi-square critical value
1	3.841
2	5.991
3	7.815
4	9.488
5	11.070
6	12.592
7	14.067
8	15.507
9	16.919
10	18.307
11	19.675